Answer to the quiz: Think of no votes (ordered) as "none of the above." Partitions of \( m \) into \( k + 1 \) non-negative integers.

\[
\binom{m+k}{k} = \binom{m+k-1}{k-1}
\]
De Morgan rules:

\[(1) \quad \left( \bigcup_{i=1}^{n} E_i \right)^c = \bigcap_{i=1}^{n} (E_i)^c \]

\[(2) \quad \left( \bigcap_{i=1}^{n} E_i \right)^c = \bigcup_{i=1}^{n} (E_i)^c \]

Proof of (1): \( x \in \bigcup_{i=1}^{n} E_i \) means: \( x \) is an element of at least one of the sets \( E_i, \ i = 1, \ldots, n \).
$x \in \left( \bigcup_{i=1}^{n} E_i \right)^c$ means: $x$ is not an element of any of the sets $E_i$, $i = 1, \ldots, n$.

That is the same thing as $x \notin E_1$, $x \notin E_2$, ...

\[ \ldots, x \notin E_m. \]

That is the same thing as $x \in (E_1)^c$, $x \in (E_2)^c$, ...

\[ \ldots, x \in (E_m)^c. \]

That is the same thing as $x \in \bigcap_{i=1}^{n} (E_i)^c$. □
This also means, if \( E_i = (F_i)^c \),

\[
\left( \bigcup_{i=1}^{m} (F_i)^c \right)^c = \bigcap_{i=1}^{m} ((F_i)^c)^c = \bigcap_{i=1}^{m} F_i.
\]

Take the complement of this identity:

\[
\bigcup_{i=1}^{m} (F_i)^c = \left( \bigcap_{i=1}^{m} F_i \right)^c
\]

This is the same as (2).
Axioms of Probability

The naive approach: If I repeat the exact same experiment \( n \) times, assume the experiment has a random outcome and the event \( E \) occurs \( n(E) \) times, then

\[
p(E) = \lim_{n \to \infty} \frac{n(E)}{n}.
\]

(1) We don't actually know if randomness exists...
If randomness actually exist, we don't know what it is.

The approach we adopt is that we assume a probability is assigned to events $E$ and we try to study its properties.

If the sample space $S$ is infinite then
not every event $E$ may have a probability. An event that has a probability is called measurable.

**Axiom 1**: $0 \leq P(E) \leq 1$

**Axiom 2**: $S$ is measurable and $P(S) = 1$

**Axiom 3**: If $E_i$ are measurable disjoint events $i = 1, 2, \ldots$ $i \neq j \Rightarrow E_i \cap E_j = \emptyset$
then \( \bigcup_{i=1}^{\infty} E_i \) is measurable and
\[
P \left( \bigcup_{i=1}^{\infty} E_i \right) = \sum_{i=1}^{\infty} P(E_i).
\]

**Axiom 4:** If \( E \) is measurable, so is \( E^c \).