1. **True or False**: the ideal $(3)$ is prime in $\mathbb{Z}/12\mathbb{Z}$. Briefly explain!

   True. By the third isomorphism theorem, $(\mathbb{Z}/12\mathbb{Z})/(3) \cong \mathbb{Z}/3\mathbb{Z}$, which is a field, hence a domain. So $(3)$ is prime.

2. In the ring $R = \mathbb{Z}/12\mathbb{Z}$, consider the multiplicative submonoid $U = R\setminus \{0, 3, 6, 9\} = \{1, 2, 4, 5, 7, 8, 10, 11\}$. Is the canonical map $R \to U^{-1}R$ injective? Briefly explain.

   No. The class $3$ is sent to zero: note that $3/1 = 0/1$ because $4(3 \cdot 1 - 0 \cdot 1) = 0$ in $\mathbb{Z}_{12}$.

3. Are there any (non-zero) ring maps $\phi : \mathbb{Z}/16\mathbb{Z} \to S$ (where $S$ is a commutative ring) sending the element $2$ to a unit? Justify your answer using the universal property of localization.

   No. If $\bar{2}$ maps to a unit under $\phi$, then so does the multiplicative monoid it generates, namely $U = \{1, 2, 4, 8, 10\}$ By the universal property of localization, because $\phi(U) \subset S^*$, we can factor $\phi$ as the composition

   $$\mathbb{Z}/16\mathbb{Z} \to U^{-1}\mathbb{Z}/16\mathbb{Z} \to S.$$ 

   But since $0 \in U$, the localization $U^{-1}\mathbb{Z}/16\mathbb{Z}$ is the zero ring, so $\phi$ is the zero map.