Let $\mathbb{F}$ be any field, and let $f$ be a polynomial in $\mathbb{F}[X]$ of degree $d > 0$. Consider the quotient ring $R = \mathbb{F}[X]/(f)$. Let $x$ denote the class of $X$ in this quotient ring.

1. Show that every element in the quotient ring can be represented by a unique element of the form
   \[ a_{d-1}x^{d-1} + a_{d-2}x^{d-2} + \cdots + a_1x + a_0, \]
   where each $a_i \in \mathbb{F}$.

2. Describe the underlying additive group structure of $R$ using expressions as in (1). Prove the structure of the underlying abelian group of $R$ is completely determined (up to isomorphism) by the degree of $f$.

3. Study the rings $R$ in the case $\mathbb{F} = \mathbb{F}_2$ and $f$ is each of the four following: $f = X^2$, $f = X^2 + X$, $f = X^2 + 1$ and $f = X^2 + X + 1$. How many elements are in each of these rings? What is the additive group structure? Write multiplication tables for these groups. Which among these are isomorphic to each other? Which are domains? Are any fields?

4. Classify all rings (up to isomorphism) of the form $R$ with $\mathbb{F} = \mathbb{F}_2$ and $d < 4$. 