

MATH 454 Homework #2

(1) Consider the heat equation

$$\frac{\partial u}{\partial t} + \beta u = \frac{\partial^2 u}{\partial x^2}$$

in the domain  $x \in [0, L]$ .  $u = u(x, t)$  is the temperature and  $\beta > 0$  The boundary conditions are:

$$u(0, t) = 0 \quad \text{and} \quad \frac{\partial u}{\partial x}(L, t) = 0$$

The initial conditions are

$$u(x, 0) = f(x)$$

Find  $u(x, t)$  for  $t > 0$ . Write down explicit solutions for  $L = \pi$  and

(a)  $f(x) = \sin(x/2)$

(b)  $f(x) = 1$

(2) Consider Laplace's equation on the square  $[0, \pi] \times [0, \pi]$ . The governing equation for  $u = u(x, y)$  is

$$\nabla^2 u = 0, \quad \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

The boundary conditions are

$$u(x, 0) = 0 \quad u(x, \pi) = f(x), \quad u(0, y) = 0, \quad \text{and} \quad u(\pi, y) = g(y) \quad (1)$$

Find  $u(x, y)$ . Evaluate your solution if  $f(x) = \sin(x)$  and  $g(y) = \sin(y)$

(3) Text page 56 question 2.3.5