

MATH 454 Homework #3

(1) Consider Laplace's equation on the square $[0, \pi] \times [0, \pi]$. The governing equation for $u = u(x, y)$ is

$$\nabla^2 u = 0, \quad \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

The boundary conditions are

$$u(x, 0) = f(x), \quad u(x, \pi) = g(x), \quad \frac{\partial u}{\partial x}(0, y) = 0, \quad \text{and} \quad \frac{\partial u}{\partial x}(\pi, y) = 0 \quad (1)$$

Find $u(x, y)$.

(a) Evaluate your solution if $f(x) = \cos(x)$ and $g(x) = \cos(7x)$.

(b) Evaluate your solution if $f(x) = a$ and $g(x) = b$, a and b are constants. Here you should notice that your answer should simplify greatly. Explain why. Write down the very simple expression for $u(x, y)$.

(2) Text page 126 question 3.4.11

(3) Text page 126 question 3.4.12

(4) Text page 131 question 3.5.1