

MATH 454 Homework #4

(1) Consider the problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + Bu$$

in the domain  $x \in [0, L]$  with  $u(0, t) = u(L, t) = 0$ . Find the critical value of  $B$ ,  $B_c$ , so that if  $B < B_c$  then  $\lim_{t \rightarrow \infty} u(x, t) = 0$

(2) Consider the modified wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - u$$

subject to the boundary conditions:

$$u(0, t) = 0 \quad \text{and} \quad u(L, t) = 0$$

Solve the initial value problem if the initial conditions are:

$$u(x, 0) = \sin \frac{\pi x}{L}$$

and

$$\frac{\partial u}{\partial t}(x, 0) = \sin \frac{2\pi x}{L}$$

(3) Consider the driven wave equation for  $x \in [0, \pi]$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 1 + \cos(x) + \sin(3t/2) \cos(3x)$$

subject to the boundary conditions:

$$\frac{\partial u}{\partial x}(0, t) = 0 \quad \text{and} \quad \frac{\partial u}{\partial x}(\pi, t) = 0$$

Solve the initial value problem if the initial conditions are:

$$u(x, 0) = 0 \quad \text{and} \quad \frac{\partial u}{\partial t}(x, 0) = 0$$