Spotting the math in *Spot it!*

Let’s play the game!!

(1) What property do pairs of cards in this deck have that makes the game work?

(2) How many cards are in the *Spot It!* deck?

(3) There are eight different pictures on each card. How many different pictures show up in the whole deck? (There is a long way to answer this question and a shorter way to answer this question).
(4) How many times does each picture appear in the deck? Is it the same answer for every picture?

(5) For manufacturing reasons, each *Spot it!* deck is missing some cards :( How many cards do you think are missing from your deck? Can you determine which cards are missing?
Building our own *Spot it!* decks. Let’s think about how to make a deck of cards that has the *Spot it!* property.

Here is an example of a *Spot it!* deck in which every card contains exactly two pictures:

![Deck example](image)

(6) The deck above contains three cards. Show that this is the largest size for a *Spot it!* deck in which every card contains two pictures.

(7) Make a *Spot it!* deck in which every card contains exactly three pictures. How many cards does your deck have? Is this the largest size for the deck? How many total pictures did you use?
(8) Is it possible to build a *Spot it!* deck in which every card contains exactly four pictures? If yes, how many cards are in an optimal deck? What is the total number of pictures required to make all of the cards in the deck?
(9) Is it possible to build a *Spot it!* deck in which every card has exactly five (seriously?!)
pictures on it? If yes, how many cards are in the deck? What is the total number of
pictures used to make the deck?
Let’s keep track of what we know so far about *Spot it!* decks:

<table>
<thead>
<tr>
<th>number of pictures per card</th>
<th>number of cards in the deck</th>
<th>total number of pictures used</th>
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<td>15</td>
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</table>

(10) Using the Finding Dory version of the game, we know that it is possible to build a *Spot it!* deck in which each card contains exactly six pictures. BUT as before, there are some cards missing from this deck. How many cards are missing? What are they?

(11) What are the patterns you see in the table?

(12) Suppose that we want to build a *Spot it!* deck in which every card contains exactly $n$ pictures. If we are successful and can build such a deck, how many cards will the deck have? What is the total number of pictures that will be used to make this deck?
Do Spot it! decks always exist? Let’s fix a positive integer $n$. Can we build a Spot it! deck in which every card contains exactly $n$ pictures? Here is what we know about the answer to this question:

**Fact 1 (using Abstract Algebra).** If $n - 1$ is equal to a power of a prime number, then a Spot it! deck exists.

**Fact 2 (using the Bruck-Ryser Theorem, 1949).** Divide $n$ by 4, and suppose that the remainder is equal to 2 or 3. If a Spot it! deck exists, then the integer $n - 1$ must be the sum of two squares\(^1\).

**Fact 3 (using a computer search\(^2\), 1989).** There is no Spot it! deck in which every card contains exactly 11 pictures.

(13) Using the facts above, and your answer to (12), complete as much of the table on the previous page as you can.

In all remaining cases, it is currently unknown if a Spot it! deck exists!

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**Fun follow up to think about later.** So far we have talked about the question of whether a Spot it! deck exists. A more general question is: given a positive integer $n$, how many different\(^3\) Spot it! decks are there in which every card contains exactly $n$ pictures?

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\(^1\)there are integers $x$ and $y$ such that $n - 1 = x^2 + y^2$

\(^2\)not everyone believes this computer search.

\(^3\)this requires knowing what it means for two decks to be different - hmmm...
**Game changing.** Let’s try to alter the game in various ways to see what happens - work with the Finding Dory game or the original game.

(14) If you remove a handful of cards from *Spot it!* and then play, does the game still work? Why or why not? (Give it a try!)

(15) Pick your favorite picture in the game. Imagine that you have erased that picture from a single card in the *Spot it!* deck. Now play the game. Does the game still work? Will it always work? Why or why not?

(16) Now imagine erasing your favorite picture from every single card in the deck. Does the game still work? Will it always work? Why or why not?

(17) Is there a way to systematically remove cards and/or pictures from the Finding Dory game to obtain a ‘smaller’ *Spot it!* deck in which each card contains exactly five pictures? It might be neat to see a smaller version of the game inside a larger version somehow. Though we expect that this might not work (see the chart).

(18) Come up with a fun question about *Spot it!* to think about :)