Some guidelines for your assignment

1. You must read and attempt the problems before meeting with your team. Even if you aren’t able to obtain all the answers, being prepared during the team meetings helps your group work more efficiently during the meeting.

2. Don’t be discouraged if you cannot solve most of the problems on your own — this is perfectly normal. This is part of why you are being assigned to work on these assignments as a group; make sure to discuss your questions and ideas with your teammates.

3. If your team is having trouble with a particular problem, try visiting the Math Lab (our math tutoring center in EH B860) with your teammates to get help.

4. Make sure everyone is involved and no-one feels excluded during the meetings. If you notice someone is shy, actively encourage them to contribute to the group!

5. Ask your teammates to explain their reasoning behind their answers if you don’t understand it. Remember that all members of the team are responsible for this assignment, and everyone should be on board with what the team turns in.

6. Write up your final solutions neatly, and make sure your explanations are clear and complete. Make sure you go over the Team Homework Tutorial on the course website:

   https://instruct.math.lsa.umich.edu/support/teamhomework/
1. A new, intense, action-packed, online game called \textit{World of Throwdown} is coming out soon. It will feature many beloved characters, including Pikablu, B.O.B., Bear and Chicken, Boss Baby, and, of course, Cabbage. Fans all over the world are incredibly excited, and publisher Wizzard are already thinking of how to cash in on it.

(a) Currently, there are two price schemes suggested. 

For scheme \textit{A} there is a $15 fee for the game, and the subscription is an additional $5 per month. However, this monthly fee will drop to $3 starting in the 7th month to retain as many players as possible.

In scheme \textit{B} the players pay $25 for the game with a monthly subscription fee of $2 a month. However, starting in the 9th month, Wizzard will raise the monthly fee to $7 a month, hoping that players are too invested to give up. 

(i) Find a piecewise function \( C = a(q) \) that models the total cost \( C \) to play the game \( q \) months according to scheme \textit{A}.

(ii) Find a piecewise function \( C = b(q) \) that models the total cost \( C \) to play the game \( q \) months according to scheme \textit{B}.

(iii) Graph the two functions on the same graph.

(b) The greedy executives of Wizzard believe they can successfully market the game so that they make the most money from each customer (i.e. They can get customers who will spend the most under scheme \textit{A} to sign up for scheme \textit{A}, and likewise for scheme \textit{B}).

(i) Find the exact intersection points of \( a(q) \) and \( b(q) \), and give a practical interpretation of what they mean in the context of this scenario.

(ii) Find a piecewise function modeling the cost needed to play the game for \( q \) months, assuming Wizzard’s marketing strategy is perfectly successful.

\[ \text{Note that any cancellations of subscription will result in a fee being charged based on the time remaining in the current month.} \]
(c) Rob, a Cabbage main, is super invested in the game. Upon the game’s release at midnight, he plays for 24 hours straight and ends up getting his character’s woodcutting skill to max level. Suppose $t$ represents the time, in hours, after 12:00 pm (noon) on the day of the game’s release, and $L(t)$ is the character’s woodcutting level at that time. A graph of $L(t)$ is given below:

Note that the graph of Rob’s level gain is similar before noon and after noon, but different pieces of the graph have been shifted around different amounts.

(i) On what intervals is the function concave up and concave down? Express your answer using interval or inequality notation.

(ii) Suppose that for $-12 \leq t \leq 0$, $L(t)$ is given by the following piecewise defined function:

$$L(t) = \begin{cases} 
B(t) & -12 \leq t < -8 \\
40 & -8 \leq t \leq -6 \\
60\sqrt{1 - \frac{t^2}{36}} + 40 & -6 < t \leq 0
\end{cases}$$

Using the graph above, write a piecewise function for $L(t)$ for $-12 \leq t \leq 12$. (Hint: Use shifts!)

(iii) What is the max woodcutting level in the game?

\footnote{This means negative $t$ values represent time before noon}
2. Zunari is a merchant who runs a profitable business in an island town, selling rolls of sailing cloth and locally grown flowers. These items are a big hit among the locals, but given the vast amount of foreign tourists to his island, he wants to expand his wares to attract their business as well. After making contact with many other merchants, the following trades are now available to him:

- He can trade $c$ rolls of cloth for $P(c)$ pinwheels
- He can trade $c$ rolls of cloth for $F(c)$ flags
- He can trade $f$ flowers for $Q(f)$ pinwheels
- He can trade $f$ flowers for $S(f)$ figurines
- He can trade $s$ figurines for $J(s)$ flags

Once Zunari has acquired these new items in his shop, he attracts many foreign buyers to his business. However, his customers like to pay with different currencies than what he used to. The two new currencies he receives are rupees and crystals. Below is what Zunari will sell his items for:

- $p$ pinwheels sell for $R(p)$ rupees.
- $ℓ$ flags sell for $M(ℓ)$ crystals.
- $s$ figurines sell for $X(s)$ rupees or $Y(s)$ crystals.

(Hint: You may want to draw a diagram to visualize all of the possible trades in this problem!)

(a) The large number of possible trades has left Zunari a little confused. He needs to know how to properly get certain items for his business to run efficiently. Assume that none of the merchants allow for tradebacks (i.e. using inverses for the functions above is not allowed in this part of the problem). Find all possible composition of trades that let Zunari do the following:

(i) Obtain flags from his initial wares (flowers/rolls of cloth).
(ii) Obtain rupees from flowers.
(iii) Obtain pinwheels from crystals.
(iv) Obtain crystals from his initial wares (flowers/rolls of cloth).

(b) After long nights of data analysis, self-loathing, and prayer, Zunari has finally compiled how some of his functions work. His findings can be found in the table to the right. Use this to answer the following questions:

\[
\begin{array}{|c|c|}
\hline
P(c) & 4c - 2 \\
F(c) & 5c - 4 \\
J(s) & 3s - 1 \\
R(p) & 17.2p - 8.7 \\
X(s) & 23.6s - 12.2 \\
M(ℓ) & 11.21ℓ - 3.13 \\
\hline
\end{array}
\]

*Assume the inputs for all functions in the table must be at least one.

(i) Find a formula for $M(F(c))$. Using this, compute and give a practical interpretation of $M(F(2))$.

(ii) Suppose Zunari wants to see how many rolls of sailing cloth it takes to trade for a certain amount of rupees. He comes up with the function $I(z) = \frac{z}{0.8} + 41$ to model how many rolls of sailing cloth he traded for $z$ rupees. Which composition of functions from the table above could $I$ be an inverse of? Is it actually an inverse?
(c) One of Zunari’s possible trades is to trade $f$ flowers for $S(f)$ figurines, which he then trades the figurines for $J(S(f))$ flags. When he writes down his records, though, he needs to backtrack and figure out how many flowers he traded away. After some thinking, he determines that if he got $x$ flags from the above trade, then he must have traded away $W(x)$ flowers. He has confirmed that $W(x)$ is actually an inverse of the composition $J(S(f))$.

(i) Give a practical interpretation of $W(8)$

(ii) Suppose that $W(x) = \frac{x}{30} + \frac{11}{15}$. Using this, find a formula for $S(f)$. 