Some guidelines for your assignment

1. You must read and attempt the problems before meeting with your team. Even if you aren’t able to obtain all the answers, being prepared during the team meetings helps your group work more efficiently during the meeting.

2. Don’t be discouraged if you cannot solve most of the problems on your own — this is perfectly normal. This is part of why you are being assigned to work on these assignments as a group; make sure to discuss your questions and ideas with your teammates.

3. If your team is having trouble with a particular problem, try visiting the Math Lab (our math tutoring center in EH B860) with your teammates to get help.

4. Make sure everyone is involved and no-one feels excluded during the meetings. If you notice someone is shy, actively encourage them to contribute to the group!

5. Ask your teammates to explain their reasoning behind their answers if you don’t understand it. Remember that all members of the team are responsible for this assignment, and everyone should be on board with what the team turns in.

6. Write up your final solutions neatly, and make sure your explanations are clear and complete. Make sure you go over the Team Homework Tutorial on the course website:

   https://instruct.math.lsa.umich.edu/support/teamhomework/
1. Rob runs a local ice cream shop that has fallen on some hard times. In order to determine why he’s losing money, he starts to track his ice cream sales after the new year. The following functions give the number of kilograms (kg) of different ice cream flavors Rob sells on the \( t \)-th day of 2019.

- \( C(t) \): Coffee
- \( M(t) \): Mint Chocolate Chip (MCC)
- \( S(t) \): Strawberry-Banana
- \( B(t) \): Black Sesame

Assume that Rob only sells these four flavors, and in parts (a) and (b) of this problem, give your answers in terms of the four functions above.

(a) Being the objectively superior flavor, MCC routinely outsells coffee each day. How many more kg of MCC ice cream does Rob sell than kg of coffee ice cream on the \( t \)-th day of 2019?

(b) What percentage of the total weight of ice cream sold on the \( t \)-th day of 2019 is coffee flavor?

(c) Rob sold 5kg of he new, and growingly popular, black sesame ice cream on the third day of 2019, and he sold 25kg on the seventh day of 2019! Find a formula for a \textbf{power function} that could model \( B(t) \) (because we are looking for a \textit{model} for the amount sold, the outputs on other days of the year need not be integers).

(d) The popularity of MCC is cannibalizing sales of strawberry-banana. Rob determines that the cube of \( S(t) \) is inversely proportional to the square root of \( M(t) \). If Rob sells 2kg strawberry-banana when he sells 36kg MCC, write a formula for \( M(t) \) in terms of \( S(t) \).

2. For this problem, please feel free to use Desmos to get ideas and check your answers.

(a) Let \( P(\ell) \) be a polynomial of degree \( m \), and \( Q(\ell) \) be a polynomial of degree \( n \), where \( m \geq n \). What is the maximum number of intersection points that the two can have? \textit{(Hint: Try to answer this for a few specific values of \( m \) and \( n \), and then try to draw a conclusion from there)}.

(b) Write a polynomial \( L(z) \) that satisfies the following properties:

- \( L(z) \) has degree 5
- \( L(z) \) only has zeros at \( z = -4, 3, 7 \)
- The absolute value of the leading coefficient is 9.6
- \( \lim_{z \to -\infty} L(z) = -\infty \)
- \( L(z) \) is increasing on the interval \([2.5, 3.5]\)
- \( L(z) \) is non-negative on the interval \([6.5, 7.5]\)

(c) Give a polynomial of lowest degree whose graph could possibly be:
3. A rational function is a function given by the quotient of two polynomial functions, i.e. a rational function \( R(x) \) is given by

\[
R(x) = \frac{A(x)}{B(x)}
\]

where both \( A(x) \) and \( B(x) \) are polynomials. For this problem, you only need what you know about combination of functions and polynomials.

(a) Many rational functions are not defined for all real numbers. Using the general form \( R(x) = \frac{A(x)}{B(x)} \), where would this rational function not be defined? What would be the domain of \( R(x) \)?

(b) Consider the rational function

\[
R(x) = \frac{x^2 + 1}{(x - 3)}
\]

This function has a vertical asymptote at \( x = 3 \).

(i) Copy and then fill in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>3.5</th>
<th>3.1</th>
<th>3.05</th>
<th>3.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Using your filled in table, explain what happens to the output of \( R(x) \) when the input approaches 3. Use this to justify why \( R(x) \) has a vertical asymptote at \( x = 3 \).

(iii) Find \( \lim_{x \to 3^+} R(x) \) and \( \lim_{x \to 3^-} R(x) \).

(c) The end behavior of a rational function \( R(x) = \frac{A(x)}{B(x)} \) is determined by the leading terms of both \( A(x) \) and \( B(x) \). More specifically, if the leading term of \( A(x) \) is \( ax^n \), and the leading term of \( B(x) \) is \( bx^m \), then

\[
\lim_{x \to \pm \infty} R(x) = \lim_{x \to \pm \infty} \frac{ax^n}{bx^m} = \lim_{x \to \pm \infty} \frac{a}{b} x^{n-m}
\]

Using what you know about end behavior of polynomials, explain why this is true.

(d) Now let \( R(x) = \frac{A(x)}{B(x)} \) be a rational function where \( A(x) \) and \( B(x) \) are two polynomials with no common zeros.\(^1\) Using what you saw above, figure out where \( R(x) \) would have zeros and vertical asymptotes. Your answer should reference the functions \( A(x) \) and \( B(x) \). Make sure to justify your response.

\(^1\) You’ll see what happens when they do have common zeros in class later in the semester.