Some guidelines for team homework

• You must read and attempt the problems before meeting with your team. Even if you aren’t able to obtain all the answers, being prepared during the team meetings helps your group work more efficiently during the meeting.

• Don’t be discouraged if you cannot solve most of the problems on your own — this is perfectly normal. This is part of why you are being assigned to work on these assignments as a group; make sure to discuss your questions and ideas with your teammates.

• If your team is having trouble with a particular problem, try visiting the Math Lab (our math tutoring center in EH B860) with your teammates to get help.

• Make sure everyone is involved and no-one feels excluded during the meetings. If you notice someone is shy, actively encourage them to contribute to the group!

• Ask your teammates to explain their reasoning behind their answers if you don’t understand it. Remember that all members of the team are responsible for this assignment, and everyone should be on board with what the team turns in.

• Write up your final solutions neatly, and make sure your explanations are clear and complete. Make sure you go over the Team Homework Tutorial on the course website:

    https://instruct.math.lsa.umich.edu/support/teamhomework/
1. Phoebe realized that her income from playing music at Central Perk is not enough to support her and Sir Whiskerson financially. Therefore, she decided to pick up more massage clients but also change the amount of money she charges them. She came up with two alternative plans:

(i) Plan A:
- Every client has to pay a fixed fee of $3 (this is to cover for some of the cost of towels, oils, hot stones etc.).
- She charges $1.50 per minute for the first 40 minutes of the session.
- She charges $1 per minute for every minute after that.

(ii) Plan B:
- Every client has to pay a fixed fee of $9 (this is to cover for some of the cost of towels, oils, hot stones etc.).
- She charges $0.50 per minute for the first 50 minutes of the session.
- She charges $2 per minute for every minute after that.

Every session lasts at most 90 minutes. Let $A(m)$ and $B(m)$ be the amount of money (in $) Phoebe charges her clients for $m$ minutes of massage for plans A and B, respectively.

(a) Evaluate $A(45)$ and $B(52)$.
(b) Find a formula for the functions $A(m)$ and $B(m)$.
(c) Plot $A(m)$ and $B(m)$ on the same graph. Make sure your graph is well-labeled.
(d) In order to help decide which plan to use, Phoebe would like to know if there is a massage session length that would cost the same under plans A and B. Can you help her?
(e) Phoebe knows that an average session lasts approximately 60 to 70 minutes. Which plan should she follow to make the most money?
(f) The function $d = A(m)$, where $d$ is the cost (in $) of a massage session that lasts $m$ minutes, has an inverse function. Write a formula for its inverse $A^{-1}(d)$.
(g) Give a practical interpretation of the expression: $A^{-1}(85) = 62$.

2. Mrs K is Ben’s new Math teacher at school. She just moved from Spain and isn’t used to speaking only in English, so sometimes she mixes in Spanish words. One day she gave the class the following exercise and her hint was: “hacia atrás”.

Graph the function $f(x)$ given the information below:

- For $x$ in [-6.5,-4] the graph of $g(x) = f(x + 3)$ has a constant rate of change which is equal to $\frac{9}{5}$.
- For $-1 \leq x \leq 3$, the graph of the function $h(x) = (4.5)^{3-x}/4$ is a shift up by 2 units of the graph of $f$. (Hint: Graph the function $h(x)$ to help find this part of $f$.)
- For $3 \leq x \leq 5$, $f(x) = f(4)$.
- For $9 \leq x \leq 13$, $k(x) = f(x-4) - 3$. The graph of $k(x)$ is shown below:
3. David and Max recently moved to Minsk for a research project in the Physics Department of the Belarusian State University. In one of their experiments they recorded the position of a particle that was moving horizontally at different times for 10 seconds.

(a) In the table below \( S \) represents the observed position of the particle (its distance from Max in inches) \( t \) seconds after the experiment began. David thinks the position of the particle can be modeled as a linear function of time. Could that be true? If yes, write a formula for \( S \) in terms of \( t \).

<table>
<thead>
<tr>
<th>( t ) (in seconds)</th>
<th>1</th>
<th>2</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S ) (in inches)</td>
<td>2</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

(b) What is the particle’s position at \( t = 10 \) according to David’s model?

(c) Max on the other hand thinks that the position of the particle should be modeled by the function:

\[
p(t) = \frac{23t^2 - 13t + 14}{(t + 2)(t + 3)}
\]

Graph the function \( p(t) \) using your calculator or some other graphing app (Desmos might be useful here.). Could \( p(t) \) model the position of the particle?

(d) What is the domain and range for both models in the context of this experiment?

(e) If the function \( p(t) \) was given to you outside of the context of this problem, what would the domain and range be?