1. Do not open this exam until you are told to do so.

2. Do not write your name anywhere on this exam.

3. This exam has 9 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.

5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.

6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.

7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.

8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

9. Turn off all phones and smartwatches, and remove all headphones and earbuds.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. [15 points] Chandler left some takeout food in the refrigerator and the food is growing bacteria. The mass of the bacteria (in mg) on the food \( t \) hours after Chandler put it in the refrigerator can be modeled by an exponential function \( N(t) \). There was 1 mg of bacteria on the food 3 hours after he put it in the refrigerator, and there were 10 mg of bacteria on the food 12 hours after he put it in the refrigerator.

Throughout this problem, be sure to include **units** where appropriate, give all answers in **exact** form, and **circle** your answer for each part.

a. [5 points] Find a formula for the exponential function \( N(t) \).

**Solution:** We know that \( N(t) = ab^t \) because \( N \) is exponential. Using the points \((3, 1)\) and \((12, 10)\), we can set up

\[
1 = ab^3 \quad \text{and} \quad 10 = ab^{12}
\]

Then \( a = 1/b^3 \) from the first equation. Substituting this into the second equation and solving gives us \( b = 10^{1/9} \). Using this expression in our equation for \( a \) gives us \( a = 1/(10^{1/9})^3 = 1/10^{1/3} \). This means

\[
N(t) = (1/10^{1/3})(10^{1/9})^t.
\]

b. [2 points] Find the initial mass of bacteria that was on the the food when Chandler put it in the refrigerator.

**Solution:** The initial mass is \( a = 1/10^{1/3} \) mg.

The number of bacteria (in mg) on Monica’s takeout food \( t \) hours after she put it in the refrigerator is given by the formula \( 2^{3t-2} \).

c. [3 points] Find the **continuous percent** growth rate of the mass of bacteria on Monica’s food.

**Solution:** Using exponent rules, we know

\[
2^{3t-2} = 2^{-2}(2^3)^t = 0.25(8)^t,
\]

so the growth factor of the bacteria is 8. This means the continuous percent growth rate is

\[
100 \ln(8)\%.
\]

d. [5 points] Find the doubling time of the mass of the bacteria on Monica’s food.

**Solution:** From the form we found above, we know that the initial amount of bacteria on Monica’s food is 0.25 mg. So we set

\[
0.5 = 0.25(8)^t
\]

and solve for \( t \). This gives

\[
t = \ln(2)/\ln(8) = \frac{1}{3}
\]

. So the doubling time is \( 1/3 \) of an hour or 20 minutes.
2. [13 points] A graph of the piecewise linear function \(a(x)\) and a table for the function \(b(x)\) are given below. Do not assume anything about the behavior of the functions beyond that which is given in the graph and the table.

\[
\begin{array}{c|c|c|c|c}
 x & -1 & 0 & 1 & 2 \\
 b(x) & 0.5 & 1 & 0.5 & 0 \\
\end{array}
\]

For each part below, find all values of \(x\) that are solutions to the given equation using the information about \(a(x)\) and \(b(x)\) above as needed. If there is no solution, or if a solution cannot be determined using the given information, write “No solution”. Show your work for each part, and circle your answer(s) for all parts of the problem.

a. [3 points] \(\frac{e^{a(x)}}{e^{b(x)}} = 1.\)

Solution:
\[
e^{a(x)} = e^{b(x)}
\]
\[
a(x) = b(x)
\]
\[
x = 0, 1, 2.
\]

b. [4 points] \(10^{a(x) - 3} = 0.01.\)

Solution:
\[
\log 10^{a(x) - 3} = \log 0.01
\]
\[
a(x) - 3 = -2
\]
\[
a(x) = 1.
\]
\[
x = 0, 2.5, 3.5, 4.5
\]

c. [3 points] \(\log (x^2 + 2x + 7) = 1.\)

Solution:
\[
x^2 + 2x + 7 = 10
\]
\[
x^2 + 2x - 3 = 0
\]
\[
(x - 1)(x + 3) = 0.
\]
\[
x = 1, -3
\]

d. [3 points] \(\ln(b(x)) = -\ln(2).\)

Solution: \(-\ln(2) = \ln(\frac{1}{2})\), so \(b(x) = \frac{1}{2}\). This means \(x = \pm 1.\).
3. [12 points] The graphs of three exponential functions, \( A(t) = Pe^{kt} \), \( B(t) = \ell t \), \( C(t) = Qm^t \) are shown below. All of \( P, Q, k, \ell, m \) are constants.

Circle **ALL** answers that apply for each part of this problem. There may be more than one answer for each part. You do not need to show your work for this problem.

a. [3 points] Which constants MUST be greater than 1?

\[
\begin{array}{cccc}
\boxed{P} & Q & k & \ell & m \\
\end{array} & \\
\text{none of these}
\]

b. [3 points] Which constants COULD NOT be greater than 1?

\[
\begin{array}{cccc}
P & \boxed{Q} & k & \ell & m \\
\end{array} & \\
\text{none of these}
\]

c. [3 points] Which constants MUST be **strictly greater** than the constant \( Q \)?

\[
\begin{array}{cccc}
P & k & \ell & \boxed{m} \\
\end{array} & \\
\text{none of these}
\]

d. [3 points] Which constants COULD be **strictly greater** than the constant \( m \)?

\[
\begin{array}{cccc}
P & Q & k & \ell \\
\end{array} & \\
\text{none of these}
\]
4. [6 points] Suppose the graph of \( y = w(x) \) contains the point \((6, -7)\). What point must be on the graph of each of the functions below? You do not need to show your work, but you may receive partial credit for correct work shown.

   a. [3 points] The graph of \( y = w(-\frac{1}{3}(x - 3)) - 4 \) must contain the point \((-15, -11)\).

   b. [3 points] The graph of \( y = -(w(x + 1) - 5) \) must contain the point \((5, 12)\).

5. [7 points] Suppose the function \( y = g(t) \) is a periodic function with period 3, domain \((-\infty, \infty)\) and range \([-1, 4]\).

   a. [4 points] Find the midline and amplitude of \( g(t) \).

      The midline of \( g(t) \) is \( y = 1.5 \).

      The amplitude of \( g(t) \) is \( 2.5 \).

   b. [3 points] Find the range of \(-\frac{1}{2}g(4t + 1) + 1\). Give your answer using interval notation.

      The range of \(-\frac{1}{2}g(4t + 1) + 1\) is \([-1, 1.5]\).
6. [13 points] Ross is designing sails for his toy sailboat in the shape of the two right triangles with one common side as shown in the figure below. The length of the segment $DC$ is 5cm. The length of the segment $AB$ is 2cm.

![Diagram of two right triangles with a common side](image)

Fill in the blanks in the following sentences. Give each answer in **exact** form using only numbers and/or trigonometric functions. You don’t need to show your work for this problem, but you may receive partial credit for work shown. Your answers should not include the symbols $\theta$ or $\alpha$.

a. [3 points] The measure of the angle $\alpha$ in **radians** is ______$\frac{41\pi}{180}$_____.

b. [2 points] The length of the line segment $BC$ is ______$5\cos(49^\circ)$_____.

c. [2 points] The length of the line segment $BD$ is ______$5\sin(49^\circ)$_____.

d. [2 points] $\tan(\theta) = \frac{5\sin(49^\circ)}{2}$.

e. [4 points] The length of the line segment $AD$ is ______$\sqrt{(5\sin(49^\circ))^2 + (2)^2}$_____.
7. [15 points] For each of the following circle the answer that correctly completes the sentence. There is only one correct answer for each part. You do not need to show any work on this problem.

a. [3 points] If \( \theta \) is an angle given in degrees satisfying \( 90^\circ \leq \theta \leq 180^\circ \), and \( \sin(\theta) = 0.8 \), then \( \cos(\theta) \) equals

- \( 0.8 \) 
- \( -0.6 \) 
- \( 0.2 \) 
- \( -0.2 \) 
- \( 0.6 \) 
- none of these

b. [3 points] The length of an arc on a circle of radius 6 inches determined by an angle of 35° is

- 210 inches 
- 35 inches 
- \( \frac{7\pi}{6} \) inches 
- \( \frac{37800}{\pi} \) inches 
- none of these

c. [3 points] If \( k \) is a positive constant, the function

\[
y = E(t) = (\pi - 1)e^{-kt} - \pi
\]

has a horizontal asymptote at

- \( y = \pi \) 
- \( y = -1 \) 
- \( y = 0 \) 
- \( y = -\pi \) 
- none of these

d. [3 points] If \( F(x) \) has a vertical asymptote at \( x = -2 \), the function \( 2F(-2x + 6) - 5 \) has a vertical asymptote at

- \( x = 10 \) 
- \( x = 0 \) 
- \( x = -6 \) 
- \( x = 4 \) 
- none of these

e. [3 points] If \( G(x) \) is an odd periodic function with domain \( (-\infty, \infty) \) and with period 4, then \( G(2) \) equals

- 0 
- 2 
- -2 
- 4 
- -4 
- not possible to find \( G(2) \)
8. [9 points] Phoebe, Rachel, and Joey are taking a road trip and their car uses \( Z(v) \) ounces of gas in one mile of driving when the speed of their car is \( v \) miles per hour. For your reference, there are 5280 feet in a mile, and there are 128 ounces in a gallon.

a. [3 points] Give a practical interpretation of the equation \( Z(28) = 10 \).

**Solution:** At a speed of 28 miles per hour, the car uses 10 ounces of gas in one mile of driving.

b. [3 points] Write an expression for the number of gallons of gas their car uses in one mile of driving while the speed of their car is 45 miles per hour. Circle your answer.

**Solution:** \( \frac{1}{128} Z(45) \)

c. [3 points] Write an expression for the number of ounces of gas their car uses in one mile of driving while the speed of their car is \( f \) feet per hour. Circle your answer.

**Solution:** \( Z \left( \frac{1}{5280} f \right) \)
9. [10 points] Suppose \( f(x) = A \cos(x) + C \) for some constants \( A \) and \( C \). Below is a table of values for \( f(x) \). Suppose the number \( b \) in the table is a positive constant less than \( \pi \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2\pi)</th>
<th>( b )</th>
<th>( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>(-4)</td>
<td>(-2)</td>
<td>( 2 )</td>
</tr>
</tbody>
</table>

In each blank below, write the correct numerical values in exact form. If it’s not possible to find a given value, write “not possible” in the blank. You do not need to show your work for this problem, but you may receive partial credit for correct work shown.

a. [2 points] \( f(0) = \) __-4__.

b. [2 points] \( C = \) __-1__.

c. [2 points] \( A = \) __-3__.

d. [2 points] \( f(-b) = \) __-2__.

e. [2 points] \( f(b + \pi) = \) __0__.