1. Do not open this exam until you are told to do so.
2. Do not write your name anywhere on this exam.
3. This exam has 9 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer, but also how you obtained it. Include units in your answer where that is appropriate.
7. You may use a TI-84, TI-89, TI-Nspire or other approved calculator. However, you must show work for any calculation which we have learned how to do in this course.
8. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
9. Turn off all phones and smartwatches, and remove all headphones and earbuds.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
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<td>10</td>
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<td>Total</td>
<td>96</td>
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</tbody>
</table>
1. [18 points] In the following table, some values of the functions $A(t)$, $B(t)$ and $C(t)$ are given.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$A(t)$</th>
<th>$B(t)$</th>
<th>$C(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>-10</td>
<td>1</td>
</tr>
</tbody>
</table>

a. [5 points] Evaluate the following expressions. If there is not enough information to evaluate an expression, write ‘NEI’. You do not need to show any work on this part of the problem but you could receive partial credit for correct work shown.

i. [1 point] $A(B(0)) =$________________________

ii. [2 points] $B(2) - B(5) =$________________________

iii. [2 points] $B(C(5) - 3) =$________________________

b. [3 points] Find all solutions to the equation $B(C(t)) = -2$.

\[ t = \]________________________

c. [10 points] For each of the following blanks write all of the functions from the table that could satisfy the given property (for example, if $A(t)$ and $B(t)$ satisfy a property, write ‘$A(t), B(t)$’ in the blank). If none of the functions could satisfy the property, write none.

i. [2 points] Which functions could be even?________________________

ii. [2 points] Which functions could be odd?________________________

iii. [2 points] Which functions could be invertible?________________________

iv. [2 points] Which functions could be periodic with period 4?________________________

v. [2 points] Which functions could be concave down on the interval $[-2, 5]$?________________________
2. [6 points] Ross and Rachel went on vacation to New Zealand. 5 hours after their plane landed in New Zealand, they went for a walk on the beach, and the tide was 5.75 feet above sea level and rising. During their trip, they noticed that the height of the tide can be modeled by a sinusoidal function of period 13 hours. If the high tide is 10 feet above sea level and low tide is 1.5 feet above sea level, give a formula for a sinusoidal function, \( h(t) \), that gives the height of the tide in feet above sea level \( t \) hours after their plane landed in New Zealand.

\[
h(t) = \text{(formula for sinusoidal function)}
\]

3. [5 points] Find the inverse of the function

\[
m = g(w) = \frac{2 + w}{1 - w}.
\]

\[
g^{-1}(m) = \text{(formula for the inverse function)}
\]
4. [12 points] Below is part of the graph of a rational function $R(x)$. The dotted lines represent vertical and horizontal asymptotes of $R$. $k$ is a positive constant.

Suppose $R(x) = \frac{P(x)}{Q(x)}$ where $P$ and $Q$ are polynomials.

a. [3 points] Find:

i. $\lim_{x \to 3^+} R(x) =$

ii. $\lim_{x \to 1^-} R(x) =$

b. [3 points] Give all values which MUST be zeros of $Q$ (note that this is asking about the zeros of $Q$, not of $R)$.

The zeros of $Q$ are ____________________.

c. [2 points] If the leading terms of $P$ and $Q$ are $3x^3$ and $2x^3$, respectively, what is the value of $k$? Circle your answer.

d. [4 points] Give formulas for both vertical asymptotes of the function $0.5R(-0.2x+1) - 6$. You don’t need to show any work on this part of the problem, but you could receive partial credit for correct work.

The vertical asymptotes of $0.5R(-0.2x+1) - 6$ are ____________________.
5. [11 points] Monica and Chandler are dieting. Below is a table for their weights, in kg, 4 and 8 weeks after they started their diet (they started at the same time).

<table>
<thead>
<tr>
<th>weeks since diet began</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monica’s weight (kg)</td>
<td>75</td>
<td>66</td>
</tr>
<tr>
<td>Chandler’s weight (kg)</td>
<td>81</td>
<td>72.9</td>
</tr>
</tbody>
</table>

a. [4 points] Find a linear function $M(w)$ that models Monica’s weight in kg $w$ weeks since she started the diet.

A possible formula for $M(w)$ is ________________.

b. [5 points] Find an exponential function $C(w)$ that models Chandler’s weight in kg $w$ weeks since he started his diet. Give all numbers in your formula in exact form.

A possible formula for $C(w)$ is ________________.

c. [2 points] Based on your models above, how much did each of them weigh when they started the diet? Give your answers in exact form.

Monica weighed ___________ kg.

Chandler weighed ___________ kg.
6. [12 points] Phoebe and Joey have started making cheese to sell at a local farmer’s market. Suppose \(P(v)\) is the weight in kg of cheese that Phoebe produces using \(v\) liters of milk. Suppose \(J(v)\) is the weight in kg of cheese that Joey produces using \(v\) liters of milk.

a. [3 points] Give a practical interpretation of the expression \(P^{-1}(2)\).

b. [3 points] Give a practical interpretation of the equation \(P(J^{-1}(3)) = 2\).

c. [3 points] Give a mathematical expression for the number of kilograms of cheese Joey produces using \(m\) mL of milk. Note that there are 1000 mL in one liter. Circle your answer.

d. [3 points] Each week at the market, Phoebe sells her cheese for the regular price of $21 per kg. Give a mathematical expression for the statement:

“If Phoebe makes cheese using 10 liters of milk, and she sells all of it at the market at regular price, she collects $126.”

Circle your answer.
7. [6 points] Give a formula for a polynomial \( T(y) \) that satisfies all of the following properties:

- \( T(-1) = 4 \).
- \( T \) has zeros at \( y = -2, 1 \).
- \( T \) has degree 3.
- \( T(y) \to \infty \) as \( y \to -\infty \).

Note that there may be more than one correct answer to this problem. Your final answer will be graded based on your correct use of notation, and based on how well your \( T(y) \) satisfies each property above.

\[
T(y) = \text{______________________________}.
\]

8. [6 points] Find all solutions \( x \) in the interval \([0, 2]\) to the equation

\[
\frac{3}{2} \cos(4(x - 1)) + 3 = 2.
\]

Give all your answers in exact form (answers not given in exact form will receive little or no credit). Your answers may include inverse trigonometric functions as needed.

\[
x = \text{______________________________}.
\]
9. [15 points] In each part of this problem circle all that apply. There might be more than one correct choice for each part.

a. [3 points] Suppose $P = 2$ when $Q = 9$ and $P = \frac{3}{2}$ when $Q = 16$. Which of the following could be true?

- $P$ could be proportional to $Q$
- $P$ could be inversely proportional to $Q$
- $P$ could be proportional to $\sqrt{Q}$
- $P$ could be inversely proportional to $\sqrt{Q}$

None of these

b. [3 points] If $x = 4$ is a solution to the equation $\tan(Bx) = 5$, where $B$ is a positive constant, then which of the following must also be a solution to the equation $\tan(Bx) = 5$?

- $x = 4 + B$
- $x = 4 - \frac{2\pi}{B}$
- $x = 4 + \pi$
- $x = \arctan(5)$

None of these

c. [3 points] Suppose the function $f$ is defined piecewise by the formula

\[
f(x) = \begin{cases} 
  x^2 & x \geq 0 \\
  3x & x < 0
\end{cases}
\]

Which of the following are solutions to the equation $f(x) = 4$?

- $x = -2$
- $x = 2$
- $x = 4/3$
- $x = -4/3$

None of these

d. [3 points] Which of the following functions are dominated by $\frac{1}{10}e^x$ as $x \to \infty$?

- $2019xe^{2019}$
- $3^x$
- $70 \ln(x)$
- $(\frac{1}{4})^{-x}$
- $2^{x+100}$

None of these

e. [3 points] Which of the following are zeros of the rational function

\[
r(x) = \frac{\pi(x - 1000)(x - 1)(3x + 5)}{(x^2 - 1)(x + 1000)}
\]

- $x = 1$
- $x = -1$
- $x = 1000$
- $x = -1000$

None of these
10. [5 points] The graph of part of a function \( m(x) \) is given below. Find a possible formula for the function. Your function can be any type of function we have learned about this semester, as long as it is the same shape as the graph below on the interval \([0, 4]\) and it goes through the indicated points. You do not need to show any work for this problem, but you may receive partial credit for correct work.

\[
\begin{array}{c}
\text{(0,1)} \quad \text{(2,3)} \quad \text{(4,1)} \\
\end{array}
\]

A possible formula for \( m(x) \) is \[ \text{_______________________________} \].