1. Do not open this exam until you are told to do so.
2. This exam has 11 pages including this cover. There are 11 problems.
3. Do not separate the pages of this exam. If they do become separated, write your name on
   every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being
   tested on this exam is your ability to interpret mathematical questions, so instructors will
   not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem,
   so that graders can see not only your answer but how you obtained it. Include units in your
   answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric
   keypad). However, you must show work for any calculation which we have learned how to
   do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch
   of the graph, and to write out the entries of the table that you use.
8. Turn off all cell phones and pagers, and remove all headphones.

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1. [8 points]

A function $f(x)$ is shown above, and graphs of various transformations of $f(x)$ are shown in the figures below. Circle the one formula beneath each of the following graphs that best describes the transformation(s) of $f(x)$.

(a) \[ (i) -f(kx), \quad k > 1 \]
\[ (ii) f(-kx), \quad 0 < k < 1 \]
\[ (iii) f(-kx), \quad k > 1 \]
\[ (iv) -f(kx), \quad 0 < k < 1 \]

(b) \[ (i) f(kx), \quad k > 1 \]
\[ (ii) f(kx), \quad 0 < k < 1 \]
\[ (iii) kf(x), \quad k > 1 \]
\[ (iv) kf(x), \quad 0 < k < 1 \]

(c) \[ (i) af(2x) + b, \quad a > 1, \quad b > 1 \]
\[ (ii) af(2x) + b, \quad 0 < a < 1, \quad b > 1 \]
\[ (iii) af(\frac{1}{2}x) + b, \quad a > 1, \quad b > 1 \]
\[ (iv) af(\frac{1}{2}x) + b, \quad 0 < a < 1, \quad b > 1 \]

(d) \[ (i) f(x + 4) - 6 \]
\[ (ii) f(x - 4) - 1 \]
\[ (iii) f(x + 4) - 1 \]
\[ (iv) f(x - 4) - 6 \]
2. [10 points] For Thanksgiving, Jane is preparing a 15-pound turkey to feed her family. The temperature, $T$, of the turkey $h$ hours after it has been put in the oven is given by

$$T(h) = 350 - 310e^{kh},$$

where $k$ is constant and $T$ is measured in degrees Fahrenheit. Use this information to answer the following questions. Show all work and remember to include units in your answer whenever appropriate.

a. [2 points] What is the initial temperature of the turkey?

b. [2 points] What is the temperature of Jane’s oven when the turkey is being cooked? (Hint: as time increases, the turkey temperature will approach the oven temperature.)

c. [4 points] If the temperature of the turkey is $180^\circ F$ in four hours, evaluate $k$. Be sure to show how you derived your answer. Express your answer exactly; approximate answers will not receive full credit.

d. [2 points] On the other side of town, John is also preparing a 15-pound turkey. He cooks his turkey in an oven that is $375^\circ F$. The temperature $J$ of John’s turkey, $h$ hours after it has been in the oven, is given by

$$J(h) = a - 335e^{-0.155h}$$

where $a$ is constant. What is the value of $a$? Explain your reasoning.
3. [9 points] The concessions stand at All-American High School finds that its weekly profit due to popcorn sales at the Friday night football game is a function of the price it charges per bag of popcorn. The weekly profit is given by

\[ W(p) = -100p^2 + 300p - 25, \]

where \( W \) is the profit made in dollars when selling popcorn at the price of \( p \) dollars per bag.

a. [3 points] Evaluate \( W(0) \). In one sentence, explain what the value of \( W(0) \) means in the context of this problem.

b. [4 points] Use the method of completing the square to determine how much the concessions should charge per bag of popcorn to earn a maximum profit. Be sure to show all appropriate work, and write your final answer in a complete sentence, including appropriate units.

c. [2 points] Justify why your answer in part (b) gives a maximum.
4. [10 points] Katie and Will want to put a fence around a rectangular garden space in their backyard. They will plant squashes and roses in their garden, and there will be a dividing fence between the two types of plants, as shown in the figure below. The dividing fence is parallel to one pair of the sides of the rectangular space. Katie and Will have 120 feet of fencing materials to use. They are not worried about the relative areas of the squash and rose gardens, but they do want to make sure that they maximize the total gardening area. Knowing that they need to provide fencing to surround the garden’s perimeter in addition to the dividing segment, what are the length and width dimensions of the total garden that will yield a maximum area for their garden? What is this maximum area?

Katie and Will’s rectangular garden space

Show all appropriate work and use units whenever appropriate.
5. [8 points] A colony of bacteria grows exponentially in a petri dish. Given that it takes 7 hours for the number of bacteria to double, how long does it take for the colony to triple in size? Analytically solve for the answer and be sure to show all work and use appropriate units in your answer.
6. [6 points] Use the data provided for \( f(x) \) to fill in the blank spaces in the following tables, given that the function represented in each table is a transformation of \( f(x) \).

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7. [4 points] The function \( j(x) \) is an odd function. Fill in as many values of the table as you can. Put an X in boxes that you cannot fill in.

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<tr>
<th>( x )</th>
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8. [15 points] Eric receives an unexpected monetary gift of $3000. Because he took a pre-calculus course in college, Eric knows that investing this money in a good bank account will allow his money to grow exponentially. After visiting several banks in Ann Arbor, he has gathered information to consider various options.

a. [3 points] At Wolverine Bank, his money will be compounded annually at a rate of 4.2%. Find a formula for \( W(t) \) that gives the balance of the account, in dollars, \( t \) years after it is invested.

b. [2 points] At TreeTown Bank, his money will earn interest at a rate of 4.15% per year, compounded monthly. Explain what it means for a balance to be compounded monthly.

c. [3 points] At TreeTown Bank, his money will earn interest at a rate of 4.15% per year, compounded monthly. Find a formula for \( T(t) \) that gives the balance of the account, in dollars, \( t \) years after it is invested.

d. [4 points] At Michigan Bank, his money will earn interest at a rate of 4.1% per year, compounded continuously. Find a formula for \( M(t) \) that gives the balance of the account, in dollars, \( t \) years after it is invested. State what the growth factor of this formula is.

e. [3 points] Which bank should Eric use to invest his $3000? Justify your answer by comparing the effective annual interest rates offered by each bank.
9. [8 points] In the imaginary land of Sontik, people live much longer than they do in the real world. As a result, the noble citizens of Sontik determine age, measured in Sontikian eras, with the relationship

\[ S = \log(R + 1), \]

where \( R \) is the age measured in Earth years and \( S \) is the age measured in Sontikian eras. Use this information to answer the following questions.

a. [2 points] What is the smallest value of \( R \) that can be used within the context of this problem? Briefly justify your answer.

b. [2 points] Citizen Alpha and Citizen Kane are two residents of Sontik. Let \( S_A \) be the age of Citizen Alpha, and \( S_K \) be the age of Citizen Kane, measured in Sontik eras. The difference between their ages is given by \( D = S_K - S_A \). Find a formula for \( D \) in terms of \( R_A \) and \( R_K \), where \( R_A \) and \( R_K \) are the ages Citizen Alpha and Citizen Kane, respectively, measured in Earth years. Include all the steps of your derivation.

c. [4 points] Suppose Citizen Kane is 2 Sontikian eras older than Citizen Alpha. Find a formula for Citizen Kane’s Earth age, \( R_K \), as a function of Citizen Alpha’s Earth age, \( R_A \). Include all the steps of your derivation.
10. [13 points] On steam-powered riverboats, the paddle wheel in the back of the boat rotates in a circular motion to propel the boat through the water. Assume the paddle wheel on the back of the riverboat Huron Queen rotates at a constant speed and suppose a sensor is placed on the very tip of the wheel. Initially, the sensor is at its highest height of 12 feet above the surface of the water. In 5 seconds, the sensor is at its lowest point, which is 5 feet below the surface of the water. Let \( H(t) \) be the height of the sensor above the surface of the water at \( t \) seconds. Positive values of \( H \) denote that the sensor is above the water, whereas negative values of \( H \) denote that the sensor is below the water’s surface.

a. [7 points] Sketch a graph of \( H(t) \) for the first 20 seconds. Be sure to label the axes appropriately, and denote the midline.

b. [2 points] What is the radius of the paddle wheel?

c. [2 points] What is the height of the center of the paddle wheel above the water? Justify your answer.

d. [2 points] How many revolutions does the paddle wheel complete in one minute? Justify your answer.
11. [9 points] Suppose \( C(A) \) is the number of cars sold by a local dealership during one month when the dealership spends \( A \) dollars on advertising.

a. [3 points] In one complete sentence, interpret \( C(500) + 20 \) in terms of number of cars and money spent on advertising.

b. [2 points] To promote employee morale, the manager of the dealership gives all of his employees a $5 bonus for each car the dealership sells. Write an expression for \( B(A) \) as a transformation of \( C(A) \), where \( B(A) \) is the monthly bonus an employee earns when the dealership spends \( A \) dollars on advertising.

c. [2 points] The manager of the dealership finds a new advertising agency that offers to cover the first $200 of advertising for free. Write an expression for \( N(A) \) as a transformation of \( C(A) \), where \( N(A) \) is the number of cars sold when the company spends \( A \) dollars on advertising with the new agency.

d. [2 points] Due to economic trouble, the dealership finds that its car sales are less than what they used to be. In fact, when the company spends \( A \) dollars on advertising, they find that the number of cars they sell in one month is 15 less than what it used to be when they spent \( A \) dollars. Find an expression for \( E(A) \) as a transformation of \( C(A) \), where \( E(A) \) is the number of cars sold during the time of economic hardship.