1. **Do not open this exam until you are told to do so.**

2. This exam has 11 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.

4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.

5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.

6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.

7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

8. **Turn off all cell phones and pagers,** and remove all headphones.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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1. [12 points] For each statement below, circle TRUE if the statement is *always* true. Otherwise circle FALSE. Justification is not expected for these problems, and no partial credit will be given.

   a. [2 points] If $t > 0$ then $(\log t)^3 = 3 \log t$.  

      True       False

   b. [2 points] The graph of $\ln(x)$ has a horizontal asymptote.  

      True       False

   c. [2 points] An investment that earns a nominal annual interest rate of $r\%$ compounded monthly has an effective annual interest rate greater than $r\%$.  

      True       False

   d. [2 points] The period of the tangent function is $\pi$.  

      True       False

   e. [2 points] An angle of 1 degree is larger than an angle of 1 radian.  

      True       False

   f. [2 points] The function $f(x) = \frac{\sin(x)}{x^2 + 1}$ is an odd function.  

      True       False
2. [12 points] You do not have to show work for these problems. However, if your answer is incorrect, work you have decided to show will be used to determine partial credit. All answers should be exact and fully simplified.

   a. [3 points] Suppose it takes 6 years for an exponentially growing bank account to grow from $1,000 to $2,000. If the growth continues at the same rate, how long will it take for the bank account to grow from $2,000 to $16,000?

   Answer: 

   b. [3 points] If the graph of \( y = e^x \) is reflected across the \( y \)-axis and vertically stretched by a factor of 2, what is a formula for the resulting graph?

   Answer: 

   c. [3 points] If \( y = -3 \) is the \( y \)-intercept for the graph of \( f(x) \), then what is the \( y \)-intercept for the graph of \( -f(3x) + 4 \)? (Answer NONE if the \( y \)-intercept cannot be determined based on the given information.)

   Answer: 

   d. [3 points] If the graph of \( y = g(x) \) has a vertical asymptote at \( x = 5 \), then what is an asymptote of the graph of \( y = g(2x - 1) + 6 \)? (Answer NONE if there is not necessarily an asymptote based on the given information.)

   Answer: 
3. [12 points] Use algebra to solve each of the following equations for $x$. You must show your work, and your answers must be exact in order to receive any credit.

a. [4 points] $10 \cdot e^{3x} = 3 \cdot 2^x$

b. [4 points] $\log(x) + \log(x - 15) = 2$

c. [4 points] $\ln(a + e^{2x}) = b$
4. [8 points]
   a. [4 points] Find a formula in vertex form for the parabola shown below. *Remember to show your work.*

   Answer: 

   b. [4 points] Find a formula for the sinusoidal function whose values (accurate to 2 decimal places) are in the following table. *Remember to show your work.*

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(t)$</td>
<td>4.25</td>
<td>-3.10</td>
<td>-10.45</td>
<td>-13.50</td>
<td>-10.45</td>
<td>-3.10</td>
<td>4.25</td>
<td>7.30</td>
<td>4.25</td>
<td>-3.10</td>
<td>-10.45</td>
</tr>
</tbody>
</table>

   Answer: $g(t) =$
5. [8 points] A particular medication has a half-life of 20 hours. A patient who has been taking this medication stops taking it.

a. [4 points] What is the continuous rate of change of the quantity of medication in this patient’s bloodstream? (Round your answer to two decimal places.)

b. [4 points] How long will it take for the quantity of medication in the patient’s bloodstream to be decreased by 90% from what it was when she stopped taking the medication? (Round your answer to two decimal places.)
6. [8 points] In the last seconds of the NCAA Men’s Basketball Championship game, A. Hero from Team Cinderella has the chance to score a winning basket. He drives down the court and goes up for the shot. The height (in feet) of A. Hero above the ground $t$ seconds after leaving the ground is given by $h = -16t^2 + 32Ct$ for some positive constant $C$.

a. [4 points] Use the method of completing the square to write the equation for $h$ in vertex form.

Answer: $h =$

b. [2 points] How high above the ground is A. Hero at the top of his jump?

c. [2 points] There are 0.8 seconds left in the game when A. Hero goes up for the shot, and he shoots the ball when he reaches the top of his jump. Team Cinderella will win the game if the game has not yet ended when he shoots. For what values of $C$ will Team Cinderella win the game?
7. [20 points]
The total amount of air in our lungs as we breathe in and out can be modeled by a sinusoidal function. An average relaxed adult male breathes (in and out) about 15 times per minute, each time breathing in and out about 500 mL of air. At the end of exhalation, about 2400 mL of air remain in the lungs.
Let $V = M(t)$ denote the volume (in mL) of air in the lungs $t$ seconds after inhalation begins.

a. [6 points] On the axes below, sketch a graph of $M(t)$ for $0 \leq t \leq 10$. Carefully label your axes and make sure the shape and important points can be clearly seen on your graph.

b. [4 points] Find the period, midline, and amplitude of $M(t)$. (Remember to include units.)

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
</table>
| The period of $M(t)$ is _______________
| The midline of $M(t)$ is _______________
| The amplitude of $M(t)$ is _______________ |
The total amount of air in our lungs as we breathe in and out can be modeled by a sinusoidal function. An average relaxed adult male breathes (in and out) about 15 times per minute, each time breathing in and out about 500 mL of air. At the end of exhalation, about 2400 mL of air remain in the lungs.

Let \( V = M(t) \) denote the volume (in mL) of air in the lungs \( t \) seconds after inhalation begins.

c. [6 points] An average adult female also breathes about 15 times per minute, but both the amount of air she breathes in and out and the amount of air remaining in her lungs after exhalation are about 22% less than that of an average male. Let \( A = F(t) \) denote the volume (in mL) of air in the lungs of an average female \( t \) seconds after inhalation begins.

\( \text{(i)} \) [4 points] Find the period, midline, and amplitude of \( F(t) \). (Remember to include units.)

Answers

The period of \( F(t) \) is

The midline of \( F(t) \) is

The amplitude of \( F(t) \) is

\( \text{(ii)} \) [2 points] Find a formula for \( F(t) \) as a transformation of \( M(t) \).

Answer: \( F(t) = \)

d. [4 points] A student whose breathing is modeled by \( M(t) \) is about to take an exam. He consciously decides to slow down his breathing, inhaling more deeply and exhaling more fully. When he does this, the average amount of air in his lungs stays the same even though the amount of air that he is breathing in and out increases. Let \( R(t) \) denote the volume of air in his lungs \( t \) seconds after he begins inhaling in this new way. Note that \( R(t) \) is still a sinusoidal function.

Circle the transformation or transformations below that are required in order to obtain the graph of \( R(t) \) from the graph of \( M(t) \).

- Vertical shift up
- Vertical shift down
- Horizontal shift to the right
- Horizontal shift to the left
- Vertical stretch
- Vertical compression
- Horizontal stretch
- Horizontal compression
8. [10 points] The apparent brightness of stars and other celestial bodies (i.e. how bright they appear to observers on earth) can be described in terms of stellar magnitude. The stellar magnitude $M$ of an object is defined by $M = -2.5 \log \left( \frac{F}{F_0} \right)$, where $F_0$ is a fixed positive constant and $F$ is a measure of the brightness of the object.

a. [4 points] Let $M_1$ and $M_2$ represent the stellar magnitudes of two objects of brightness $F_1$ and $F_2$, respectively. Use properties of logarithms to find a simplified formula for the difference $M_2 - M_1$ of the two stellar magnitudes in terms of $F_1$ and $F_2$.

b. [3 points] The stellar magnitude of the sun is approximately -26.73 and the stellar magnitude of a full moon is approximately -12.6. How much brighter is the sun than the full moon? (Round your answer to the nearest integer.)

c. [3 points] (This problem does not depend on the answers to parts (a) and (b) above.)
Let $h(F) = -2.5 \log \left( \frac{F}{F_0} \right)$ be the stellar magnitude of an object of brightness $F$ as above. If we define a new function $g(F) = \log(F)$, what transformations should we perform on the graph of $g(F)$ in order to get the graph of $h(F)$? (List the transformations in the order in which they should be performed.)
9. [10 points] The figure below shows a circle of radius 5 units centered at the point (3, 5).

![Circle Diagram]

a. [1 point] Find the coordinates of the point $T$.

Answer: $T = (\quad , \quad )$.

b. [3 points] Find the coordinates of the points $P$ and $Q$ in terms of $\phi$.

Answers: $P = (\quad , \quad )$ and $Q = (\quad , \quad )$.

c. [6 points] An ant begins at the point $T$ and travels counterclockwise around the circle. Let $h(d)$ be the $x$-coordinate of the ant after it has traveled a distance of $d$ units along the circle.

(i) [2 points] Find $h(5\pi)$.

Answer: $h(5\pi) =$

(ii) [4 points] Find a formula for $h(d)$.

Hint: You might find it helpful to make a rough sketch of the graph of $h(d)$.

Answer: $h(d) =$