1. Do not open this exam until you are told to do so.
2. This exam has 11 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. Turn off all cell phones and pagers, and remove all headphones.

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<th>Problem</th>
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1. [6 points]

The graph of \( f(x) \) is shown in the figure above. In each of the figures below, the graph shown represents a transformation of \( f(x) \). Note that the scale of the axes is the same in each of the graphs below as it is in the graph of \( f(x) \) above.
For each figure, circle the one formula which best represents the given transformation. Justification is not expected for these problems.

\[
\begin{array}{lll}
i. & f(x + 2) - 2 & \text{ i. } 0.5f(x) - 4 & \text{ i. } f(-0.5x) + 2 \\
ii. & f(x - 1) - 1 & \text{ ii. } f(-2x) - 2 & \text{ ii. } f(-2x) + 1 \\
iii. & 2f(x - 1) & \text{ iii. } -0.5f(x) & \text{ iii. } -f(-x) + 4 \\
\end{array}
\]
2. [14 points] Note that calculator approximations will NOT receive credit when an exact answer is required.

a. [5 points] Solve for \( t \) exactly in the equation \( 3e^t = 5^t \). (Show your work step-by-step.)

Answer: \( t = \underline{\text{}} \)

b. [4 points] Suppose \( f \) is an even function and \( f(2) = -3 \). Find the coordinates of two points that must lie on the graph of \( 2f(x - 1) \).

Answers: \( \underline{\text{}} \) and \( \underline{\text{}} \)

c. [5 points] Solve for \( t \) exactly in the equation \( \frac{\log(t^3) + \log(t^4)}{\log(1000t)} = 6 \). (Show your work step-by-step.)

Answer: \( t = \underline{\text{}} \)
3. [12 points] The “Chicago Wheel” was designed by George Washington Gale Ferris, Jr. for the World’s Columbian Exposition in Chicago. It first opened to the public on June 21, 1893. With a diameter of 264 feet, the Chicago Wheel took 9 minutes to complete a full revolution. The boarding platform was located at the bottom of the wheel, which was 6 feet above the ground. A local student from the newly established University of Chicago saw Mr. Ferris taking a test ride and decided to study the motion of this machine. Let \( H = F(t) \) be the height, in feet, of Mr. Ferris above the ground \( t \) minutes after passing the boarding platform at the bottom of the wheel.

a. [6 points] Find the period, amplitude, and midline of \( H = F(t) \). (Include units.)

Period: ______________________

Amplitude: ____________________

Midline: ______________________

b. [6 points] Use your answers from part (a) to carefully sketch a graph of \( H = F(t) \) for \( 0 \leq t \leq 18 \) on the axes below. (Remember to label the axes and make sure that important features of the graph are clear.)
4. [10 points] On December 25\textsuperscript{th}, 2009, at exactly midnight, Mr. S. Clause left his house to run some errands. Unfortunately, he forgot to shut his door all the way! When he left his house, the temperature was a comfortable temperature, but every two hours while he was away, the temperature, in °F, inside his house decreased by exactly 17 percent. (Mr. Clause lives in a rather magical place, where measurements really are exact.)

   a. [7 points] After how many hours did the temperature in Mr. Clause’s house reach 25 percent of what it was when he departed? (Show your work step-by-step and give your answer in \textit{exact form}. A calculator approximation will NOT receive credit.)

Answer: ________________________________

   b. [3 points] What was the continuous rate of decay of the temperature of Mr. Clause’s house? (Give your answer in \textit{exact form} or round to the nearest 0.01%.)

Answer: ________________________________
5. [14 points] A University of Michigan social scientist observes that the number of people, in thousands, on Central Campus on a weekday can be approximated by the function

\[ c(t) = 4 + 4t - \frac{1}{5}t^2 \]

where \( t \) is the number of hours after 5 am and \( 0 \leq t \leq A \) for a fixed constant \( A \).

a. [7 points] Rewrite the expression for \( c(t) \) in vertex form. (Use algebra and show your work step-by-step.)

Answer: \( c(t) = \)

b. [4 points] At what time are the most people on campus? (Give your answer as a time of day, e.g. “5 am” rather than “\( t = 0 \)”). How many people are on campus at that time?

Answers: time: ____________ number of people: ____________

c. [3 points] The scientist’s formula predicts that everyone will have left campus \( A \) hours after 5 am. Solve for \( A \).

Answer: \( A = \)
6. [10 points] The average high temperature in Ann Arbor oscillates in a sinusoidal fashion. On January 15\textsuperscript{th}, the average high temperature is 30\degree F, and the average high temperature increases until it reaches a maximum of 84\degree F on July 15\textsuperscript{th}. Let $T(m)$ be the average high temperature in Ann Arbor (in °F) $m$ months after January 15\textsuperscript{th}. (Note that July is six months after January.)

   a. [6 points] Find a formula for $T(m)$.

   Answer: $T(m) =$ 

   

   b. [4 points] Use your formula from part (a) to estimate the average high temperature in Ann Arbor on November 15\textsuperscript{th}. (Show your work, and round your answer to the nearest 0.1\degree F.)

   Answer: 


7. [10 points] A mathematician named Professor Anna Erdos drinks at least 8 ounces of coffee every day. In fact, she finds that the more coffee she drinks per day, the more theorems she is able to prove that day. If $T$ represents the average number of theorems Anna can prove in a day if she drinks $x$ ounces of coffee, Anna discovers that

$$T = h(x) = \log(x^a) - 8$$

for some constant $a$.

a. [6 points] Anna is able to determine that $h(2x) = h(x) + 3$. Use this information to find the value of the constant $a$. (Give your answer in exact form.)

Answer: $a =$

b. [4 points] Use your answer from part (a) to calculate how much coffee Anna would need to drink each day to produce an average of 3.5 theorems per day. (Round your answer to the nearest 0.01 ounce.)

Answer: ____________________________
8. [12 points]
A local elementary school decides to start a food drive, and people are able to donate either food or money to the school. At the end of each month the school takes all of the money that has been donated and buys food with it. The school then combines the food it was able to buy with the food that was donated that month and gives it all to a local food bank. Let \( f(D) \) be a function which gives the number of pounds of food that the school can buy with \( D \) dollars.

In the first month, \( D_0 \) dollars were donated to the school, and no food was donated. Thus, the school bought \( f(D_0) \) pounds of food. Since no additional food was donated, the school donated a total of \( f(D_0) \) pounds of food to the food bank in the first month.

For each of the situations below, use the above information to write an expression for the number of pounds of food donated in the given month. (Your answers may involve \( f \) and/or \( D_0 \).)

a. [3 points] In the second month, the school was given $1000.00 more than in the first month, and, in addition, 20 pounds of food were donated to the school.

Answer: In the second month, the school donated a total of \( \underline{\text{answer}} \) pounds of food to the food bank.

b. [3 points] In the third month, the school was given 13 pounds of food along with enough money to buy twice as much food as they bought in the first month.

Answer: In the third month, the school donated a total of \( \underline{\text{answer}} \) pounds of food to the food bank.

c. [3 points] In the fourth month, Zingerman’s decided to help with the food drive. They agreed to donate an additional $3.00 for every dollar the school raised. The school raised $100.00 more than in the first month. This month no additional food was donated.

Answer: In the fourth month, the school donated a total of \( \underline{\text{answer}} \) pounds of food to the food bank.

d. [3 points] In the fifth month, only a third as much money was donated as in the first month. Luckily, 50 pounds of food was also donated.

Answer: In the fifth month, the school donated a total of \( \underline{\text{answer}} \) pounds of food to the food bank.
9. [12 points] Three runners are racing counterclockwise on a circular track. The race is to determine who can run the farthest in 10 minutes. An observer is standing in the center of the racetrack to record how many times each runner goes around the track. The first runner is in the first lane of the racetrack, which is 35 meters from the center of the track. The second runner is in the second lane, which is 38 meters from the center of the track, and the third runner is in a third lane, which is 41 meters from the center of the track. (See diagram below.)

\[ \text{Observer} \]
\[ \text{Starting line} \]
\[ 35 \text{ m} \]
\[ 50 \text{ m} \]
\[ 9 \text{ m} \]
\[ \text{Road} \]

\[ a. \quad [3 \text{ points}] \] The observer notes that in 10 minutes, the first racer went around the track \(10 \frac{2}{3}\) times. How far did the first racer run? (Give your answer in exact form or round to the nearest 0.01 meter.)

\[ \text{Answer: } \]

\[ b. \quad [4 \text{ points}] \] The second racer ran 2402 meters in 10 minutes. (Recall that the second racer is in the second lane, which is 38 meters from the center of the track.) The observer spins while watching her run, always facing in her direction. Through what total angle (in radians) does the observer turn? (Give your answer in exact form or round to the nearest 0.01 radian.)

\[ \text{Answer: } \]

This problem continues on the next page.
This is a continuation of the problem from the previous page. The original problem statement is included here for your convenience.

Three runners are racing counterclockwise on a circular track. The race is to determine who can run the farthest in 10 minutes. An observer is standing in the center of the racetrack to record how many times each runner goes around the track.

The first runner is in the first lane of the racetrack, which is 35 meters from the center of the track. The second runner is in the second lane, which is 38 meters from the center of the track, and the third runner is in a third lane, which is 41 meters from the center of the track. (See diagram below.)

\[ \text{Observer} \]
\[ \text{Starting line} \]
\[ \text{Road} \]

\[ 35 \text{ m} \]
\[ 9 \text{ m} \]
\[ 50 \text{ m} \]

\( \text{c. [5 points]} \) There is a road located 50 meters south of the center of the racetrack and going from east to west. Assume the starting line is located at the east end of the track (see diagram above). The third racer runs at a constant speed for the entire 10 minutes and goes around the track exactly 10 times in those 10 minutes. (Recall that the third racer is in the third lane, which is 41 meters from the center of the track.)

Find a formula for \( D \), the distance in meters of the third racer from the road, as a function of \( t \), the time in minutes since she started running. (Note that \( D \) is the distance perpendicular to the road, so \( D = 50 \) when the third runner is at the starting line and \( D = 9 \) when the third runner is at the south end of the track.)

Answer: \( D = \) ___________________________