Math 105 — Second Midterm
March 22, 2011

Name: ____________________________________________  Instructor: __________________________  Section: __________________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers,** and remove all headphones.

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1. [10 points] Answer “True” or “False” for each of the following.

i) If \( \log(A) = x \) and \( \log(B) = y \), then \( \frac{\log(A)}{\log(B)} = x - y \).

   True  False

ii) The function \( m(n) = 2n^3 + n + 3 \) is an odd function.

   True  False

iii) A population of bacteria that is increasing by 25% per year has a doubling time of a little over 3 years.

   True  False

iv) If a quantity is decreasing by 18% per year, the continuous percent change is \(-19.85\%\).

   True  False

v) The arc length spanned by an angle of 30° in a circle of radius 8 cm is 240 cm.

   True  False
2. [12 points] Solve the following equations for $x$. Give the exact solution(s).

a.) $700 = 20(1.7)^{5x}$

b.) $15(2.3)^{3x} = 6(4.9)^x$

c.) $64e^{(6x+3)} = 12$

d.) $\cos(x) = \frac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$
3. [12 points]

The graph of $y = g(x)$ is shown in the figure above. For each graph below, circle the formula which represents the transformation of $g(x)$.

The graph of $g(x)$

a. $-2g(x)$  
   b. $-g(2x)$  
   c. $g(-x/2)$  
   d. $-g(x)$

a. $g(-x + 1)$  
   b. $g(-x) + 1$  
   c. $-g(x) + 1$  
   d. $-g(x + 1)$

a. $-g(x)$  
   b. $g(-x)$  
   c. $-g(-x)$  
   d. $g(-x - 1)$

a. $g(x) + 1$  
   b. $g(x + 1)$  
   c. $g(x) - 1$  
   d. $g(x - 1)$
4. [12 points] The graph of \( f \) contains the point \((-1, 4)\). What point must be on the following transformed graphs?

a.) The graph of \( f(x - 4) \) must contain the point ________________

b.) The graph of \( f(x) - 2 \) must contain the point ________________

c.) The graph of \( f(x + 2) + 3 \) must contain the point ________________

d.) The graph of \( f(-x) \) must contain the point ________________

e.) The graph of \(-2f(x)\) must contain the point ________________

For the following, if the graph of \( f \) contains the point \((-1, 4)\), what other point must also be on the graph of \( f \)?

f.) If \( f \) is an odd function, the graph of \( f \) must also contain the point ________________

5. [5 points] The U-M hockey and basketball teams have made it to the NCAA playoffs. (Congratulations!) In a moment of exuberance, Carl Hagelin and Darius Morris went to the field of the Big House, and as Darius dropped a basketball, Carl hit it straight into the air with his hockey stick. For \( t \) in seconds, the height of the basketball in feet above the ground is given by the formula

\[
h(t) = -16t^2 + 47t + 3.\]

Use algebra to convert the function \( h \) to vertex form (showing your work!), and give the time that the ball reached its maximum height and the maximum height that the ball reached.
6. [20 points] In April 2009, Frito-Lay introduced a biodegradable bag for its SunChips brand snacks. Ivana B. Green is trying to compost the SunChips bags in her home compost pile. The temperature of her compost pile varies in a sinusoidal fashion over a 24 hour period with a low temperature at 4 AM of 70° F and a high temperature at 4 PM of 140° F. Let $T$ be the temperature in Ivana’s compost pile as a function of $t$, the number of hours after 4 AM.

a. [4 points] On the axes below, graph two periods of the function $T$ starting with $t = 0$. Carefully label important points on your graph.

b. [7 points] Find a formula for the function $T$, and give its amplitude, period, and midline.

$$T(t) = \text{__________________________}$$

Amplitude: \Text{__________________________}

Period: \Text{__________________________}

Midline: \Text{__________________________}

Problem continued on next page. Carefully copy your formula for $T(t)$ to the next page....
Problem 6. continued...

Write your formula for $T(t)$ from part b. here:  

\[ T(t) = \frac{300}{1 + e^{-kt}} \]

\[ c. \ [7 \text{ points}] \text{ At approximately what time(s) during the day is the temperature in the compost pile equal to } 130^\circ \text{ F? [Note: To receive full credit for this problem, you must show your work algebraically. Also note that although this problem contains degrees (temperature) as an output, your calculator should still be in } \text{radian mode.]} \]

\[ d. \ [2 \text{ points}] \text{ In order for the SunChips bag to decompose, the temperature in the compost pile must be at least } 130^\circ \text{ F for 8 hours or more in each 24 hour period. Should Ivana compost the bags in her home composting pile? Explain why or why not.} \]
7. [10 points] Ivana’s neighbor, Red Wiggler, uses worms in a process called vermicomposting to compost his organic waste. He finds that the SunChips bags biodegrade in his compost pile with a half-life of 5 weeks. On June 1st, Red started with 2.7 grams of bag material in his compost pile.

   a. [3 points] Find a formula for $Q$, the amount of SunChip bag material in Red’s compost pile, as a function of $t$, the number of weeks after June 1st.

   b. [2 points] What is the continuous growth rate of the SunChip bag material in Red’s compost pile?

   c. [2 points] By what percent does the amount of bag material in the compost pile decrease each week?

   d. [3 points] When will the amount of bag material in Red’s compost pile be less than 0.1 gram?
8. [9 points] Red Wiggler is so excited about his compost that he decides to plant a garden. The
deer in his neighborhood are quite a nuisance, so he needs to fence the garden. He decides to
use his garage as one side of the garden and fence the other three sides. He has 12 meters of
fencing material for the other three sides. His garden will be rectangular-shaped.

a. [3 points] Draw a picture of the fenced in area, labeling the width $w$ and the length $l$.

b. [3 points] Use the information that Red has 12 meters of fencing to find a formula for the
area, $A$, of the garden as a function of one variable (either $w$ or $l$).

c. [3 points] Determine the maximum area that Red can enclose. [Justify your answer with
algebra—full credit will not be given for a graph of $A$ or a table of values.]
9. [10 points] Red finds that the amount of pleasure that he feels looking at his garden depends on the number of minutes he spent working in the garden the previous day. In order to better understand this relationship, he decided to record data throughout one growing season. Each day, he measured the pleasure he felt on a scale from 0 to 100 (with 0 being totally bummed out and 100 being nearly ecstatic) and recorded the number of minutes he spent working on his garden. At the end of the growing season, Red found an invertible function $P$, which outputs the level of pleasure he feels when looking at his garden for a given $t$, the number of minutes he spent working on his garden the previous day. He also found that he spent an average of $a$ minutes a day working on his garden. For each of the following statements, pick one of the expressions (A) - (O) which best represents the statement.

(A) $P(0)$  
(B) $P^{-1}(0)$  
(C) $P(100)$  
(D) $P^{-1}(100)$  
(E) $P(a/2)$  
(F) $P(a)/2$  
(G) $P(2a)$  
(H) $2P(a)$  
(I) $P(a + 1)$  
(J) $P(a) + 1$  
(K) $P(a + 60)$  
(L) $P(a) + 60$  
(M) $2a$  
(N) $P^{-1}(2P(a))$  
(O) $P^{-1}(2a)$

a.) The level of pleasure felt the next day when Red spent twice the average number of minutes working in his garden.

b.) The level of pleasure Red felt when he spent no time the previous day working on his garden.

c.) The number of minutes Red needs to spend working on his garden to feel nearly ecstatic the next day.

d.) The level of pleasure Red feels the next day if he works on his garden one hour longer than on average.

e.) The number of minutes Red needs to spend working on his garden to feel twice the pleasure Red feels when working the average number of minutes.