Math 105 — Final Exam
April 25, 2011

Name: ____________________________________________
Instructor: ___________________________  Section: ____________

1. **Do not open this exam until you are told to do so.**

2. This exam has 9 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.

4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.

5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.

6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course.

7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

8. **Turn off all cell phones and pagers**, and remove all headphones.

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<thead>
<tr>
<th>Problem</th>
<th>Points</th>
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1. [12 points] Circle “TRUE” or “FALSE” for each of the following problems. Circle “TRUE” only if the statement is *always* true.

a.) If a chemical substance has a half-life of 2 months, then 1/4 of the substance decays in one month.

   True  False

b.) The graph of $y = \ln x$ has a horizontal asymptote.

   True  False

c.) If $\cos(\theta) = 0.2753$ and $0 \leq \theta \leq \pi/2$, then $\cos(\pi - \theta) = -0.2753$.

   True  False

d.) If $f(x) = \frac{1}{x}$, then $f(x + h) = \frac{1}{x} + \frac{1}{h}$.

   True  False

e.) If $f(x) \rightarrow 2$ as $x \rightarrow \pm\infty$, then the graph of $f$ cannot cross the line $y = 2$.

   True  False

f.) The area of a circle is $A = \pi r^2$ and the circumference of a circle is $C = 2\pi r$, so the area of a circle may be written as $A = \frac{C^2}{4\pi}$.

   True  False
2. [18 points] Solve the following. Show your work

a.) Find the average rate of change between \( x = 2 \) and \( x = 5 \) of \( g(x) = x^3 + 4x + 2 \).

b.) If \( w \) is inversely proportional to the square root of \( v \) and \( w = 8 \) when \( v = 4 \), find a formula for \( w \) in terms of \( v \).

c.) Find the yearly growth rate of \( Q \) if, for \( t \) in years, \( Q(t) = 15(e^{0.175t}) \).

d.) Find the minimum of \( h(x) = 5x^2 - 2x + \frac{36}{55} \).

e.) For \( 0 \leq \theta \leq \pi \), solve \( 2\sin(2\theta) = \sqrt{3} \). Show your answer(s) in exact form—i.e., not a decimal approximation.

f.) What arc length is spanned by an angle of \( \frac{60}{\pi} \) degrees in a circle of radius of 6 cm?
3. [9 points] Write the letters corresponding to all of the correct answers to the following questions on the line given. There may be more than one correct answer listed.

a) The average high temperatures in Ann Arbor range from a low of 30°F in January to a high of 83°F in July. Which of the following are possible formulas to model the A² temperatures? Assume that \( t = 0 \) is January 1st and that \( t \) can represent days, weeks, or months.

\[
\begin{align*}
(A) & \quad T(t) = 26.5 \sin \left( \frac{\pi}{6} (t - 3) \right) + 56.5 \\
(B) & \quad T(t) = -26.5 \cos \left( \frac{2\pi}{52} t \right) + 56.5 \\
(C) & \quad T(t) = -26.5 \sin \left( \frac{\pi}{6} t \right) + 56.5 \\
(D) & \quad T(t) = 26.5 \cos \left( \frac{\pi}{6} (t - 6) \right) + 56.5
\end{align*}
\]

The possible formulas are ________________.

b) A population of bacteria is growing by 50% per day. Which of the following statements are true?

\[
\begin{align*}
(A) & \quad \text{The population doubles in two days.} \\
(B) & \quad \text{One model for the population would be } P(t) = P_0 e^{0.4055t}. \\
(C) & \quad \text{Four days after measuring the initial population, there would be approximately 5 times as many bacteria.} \\
(D) & \quad \text{The population could be modeled by the function } P(t) = P_0 (0.50)^t.
\end{align*}
\]

The true statements are ________________.

c) In Ivana B. Green’s garden, the total area covered by weeds, \( w \) in square inches, is a function of the number of cubic inches of mulch, \( m \), that Ivana spread on her garden last fall. Thus, \( w = f(m) \). Assume that the function \( f \) is invertible. Which of the following statements are true?

\[
\begin{align*}
(A) & \quad \text{The expression } f(200) \text{ means that Ivana spread 200 cubic inches of mulch on her garden and had some number of square inches of weeds growing.} \\
(B) & \quad \text{If } f(200) = 50, \text{ then } f(f^{-1}(50)) = 200. \\
(C) & \quad \text{The expression } f^{-1}(a) \text{ represents the number of cubic inches of mulch Ivana applied if she ended up with } a \text{ square inches of weeds in her garden.} \\
(D) & \quad \text{If the total area covered by Ivana’s garden is 100 square feet, then the domain of } f^{-1} \text{ is } 0 \leq x \leq 14,400. \text{ (There are 12 inches in a foot.)}
\end{align*}
\]

The true statements are ________________.
4. [12 points] Ivana B. Green’s neighbor, Red Wiggler, has found that his composting worms are efficient, but they also multiply quickly.

   a. [3 points] The population of composting worms (ironically, called “red wigglers!”) has a doubling time of 2 months. If Red started with 1000 wigglers, find a formula for the number of wigglers Red will have in $t$ months.

   b. [3 points] According to your formula, in how many months will Red have 6000 wigglers?

   c. [6 points] Ivana is very intrigued by Red’s worms, and she wants to start her own red wiggler composting. Red has agreed to give Ivana some worms, and she wants to construct her own composting bin. She plans to make a box, and she wants the width of the box to be twice the height and the length of the box to be 2 feet longer than the width. Find a formula for the volume of the box, and determine the dimensions of the box, given that the volume should be 8 cubic feet. (Hint: draw a picture, and write a formula in terms of one variable.)
5. [13 points] Ivana is experimenting with a mixture of liquid fertilizer for her garden. She began with two liters of mixture, containing 200 ml of fertilizer. As she adds fertilizer, the concentration of fertilizer in the mixture changes.

a. [3 points] Find a formula which represents the concentration, \( C = f(x) \), of fertilizer in the mixture when Ivana adds \( x \) ml of fertilizer. (Note: 1 liter = 1000 ml.)

b. [3 points] Find and interpret, in the context of this problem, \( f(100) \).

c. [4 points] Find a formula for \( f^{-1}(C) \).

d. [3 points] Find and interpret, in the context of this problem, \( f^{-1}(0.25) \).
6. [5 points] Ivana finds that unlike Red’s wigglers, her worm population is not growing exponentially. She believes that instead, the population can be modeled by a power function. Two weeks after she started composting with worms, she had 4000 worms, and six weeks after she started composting she had 4943 worms. Find a function, $f$, which gives the number of worms in Ivana’s composting bin as a function of $t$, the number of weeks after she started composting with worms.

$$f(t) = \ldots$$

7. [5 points] Find a formula for the periodic function, $f$, that is graphed below.

$$f(x) = \ldots$$
8. [9 points] Let \( g(x) = \frac{1}{x + 3} + \frac{2}{x - 4} \).

   a. [4 points] Rewrite \( g \) as a rational function—i.e., a quotient of two polynomial functions.

   b. [5 points] Complete the following statements about the graph of \( y = g(x) \) using your formula for \( g \). If appropriate, write “NONE.”

   (i) The function \( g \) has zero(s) at ____________________________

   (ii) The graph of \( g \) has vertical asymptote(s) at ____________________________

   (iii) The graph of \( g \) has horizontal asymptote(s) of ____________________________

   (iv) The graph of \( g \) has \( y \)-intercept(s) at ____________________________

9. [5 points] The graph of a polynomial \( y = h(x) \) is given in the figure below. Determine a possible formula for \( h \).

\[
h(x) = \text{______________________________}
\]
10. [12 points] The following table and graph describe functions $f$ and $g$. Assume that the function $f$ is invertible and that the table and graph fully describe the behavior of the functions. Let $h$ be a linear function through the point $(1, 2)$ with slope $-2$. Use the functions $f$, $g$, and $h$ to answer the questions below. Show your work.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
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</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>$-2$</td>
<td>$-5$</td>
<td>$-8$</td>
<td>$-12$</td>
</tr>
</tbody>
</table>

a.) What is the value of $f(g(3))$?

b.) What is the value of $f^{-1}(-2g(-2))$?

c.) What is the value of $f(0)g(0)h(0)$?

d.) Which transformations need to be performed on the graph of $g$ to get the graph of $h(g(x))$?

e.) Estimate all solutions to the equation $h(x) - g(x) = 0$. 