

MATH 105 — FIRST UNIFORM EXAM

DEPARTMENT OF MATHEMATICS
University of Michigan

February 12, 2003, 6 pm - 7:30 pm

NAME: _____

ID NUMBER: _____

SIGNATURE: _____

INSTRUCTOR: _____

SECTION NO: _____

General Instructions: Do not open this exam until you are told to begin. The exam consists of 10 questions on 8 pages (including this cover). The exam has a 100 point scale.

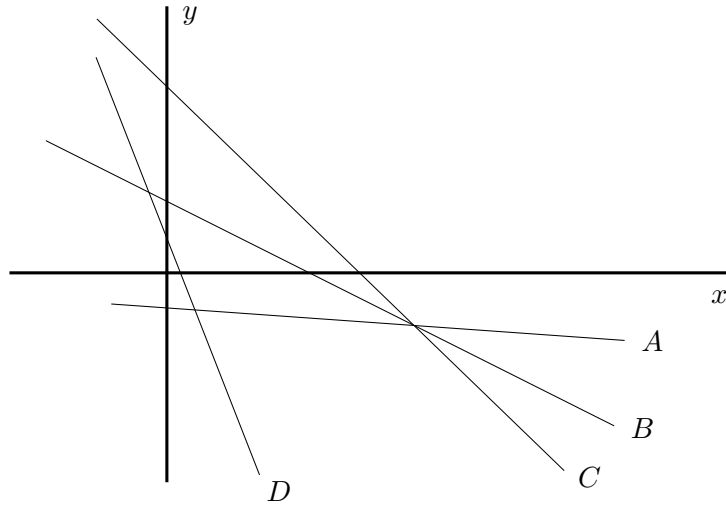
Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.

Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. Unless explicitly stated, no credit will be given for answers that do not show how they are derived. If you use graphs or tables to obtain an answer, be certain to provide an explanation (and a sketch of the graph, if that is the method) to make it clear how you arrived at your solution. Use units where appropriate.

You may use your calculator, but no other outside materials.

PROBLEM	POINTS	SCORE
1	6	
2	6	
3	18	
4	6	
5	6	
6	4	
7	21	
8	12	
9	12	
10	9	
TOTAL	100	

1. Assuming that the equations for A , B , C and D graphed below can be written in the form $y = mx + b$, use the graphs to answer the following questions.



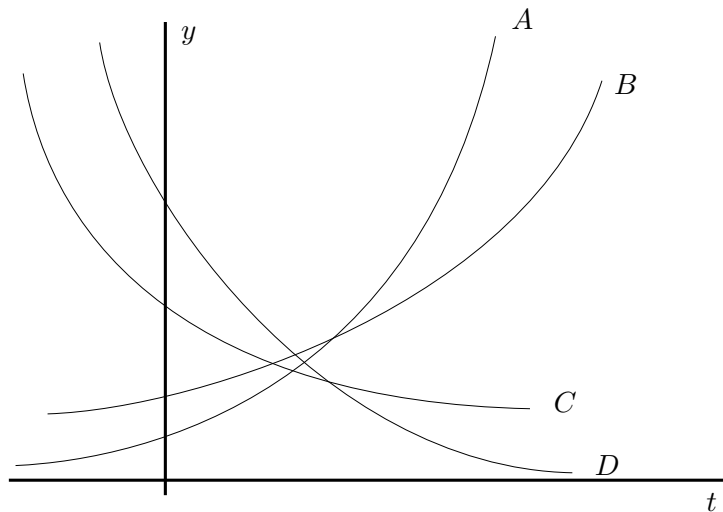
a) (3 points) Which graph gives the equation with the LARGEST value of b ?

Answer: C

b) (3 points) Which graph gives the equation with the LARGEST value of m ?

Answer: A

2. Assuming that the equations for A , B , C and D graphed below can be written in the form $y = a(b)^t$, use the graphs to answer the following questions.



a) (3 points) Which curve has the equation with the SMALLEST value of a ?

Answer: A

b) (3 points) Which curve has the equation with the SMALLEST value of b ?

Answer: D

3. A town has a population of 3000 people at year $t = 0$. Write a formula for the population, P , of the town t years later in each of the following situations. Make sure that each formula contains both of the variables and an equals sign.

a) (4 points) The town grows by 200 people per year.

$$P = 3000 + 200t.$$

b) (5 points) The town grows by 6% per year.

$$P = 3000 \cdot (1.06)^t.$$

c) (3 points) The town shrinks by 50 people per year.

$$P = 3000 - 50t.$$

d) (3 points) The town shrinks by 4% per year.

$$P = 3000 \cdot (.96)^t.$$

4. (6 points) If $(2a, b)$ lies on the graph of the line $y = 3x + 4$, what is the value of a ? Circle your answer.

a) $\boxed{\frac{b-4}{6}}$

b) $\frac{b}{6} - 4$

c) $\frac{b}{6} + 4$

d) $\frac{3b+4}{2}$

e) $6a + 4$

5. (6 points) Solve the following equation for t :

$$e^{0.044t} - 6 = 0.$$

$$2e^{0.044t} = 6$$

$$\ln e^{0.044t} = \ln 6$$

$$0.44t = \ln 6$$

$$t = \frac{\ln 6}{0.044}$$

6. (4 points) The thrust, T , in pounds delivered by a ship's propeller is proportional to the square of the propeller speed, R , in rotations per minute, times the fourth power of the propeller diameter, D , in feet. What happens to the thrust if the propeller diameter is doubled but the propeller speed doesn't change?

The given relationship is $T = C \cdot R^2 \cdot D^4$, for some constant C . By doubling the diameter, we find the new thrust is $T' = C \cdot R^2 \cdot (2D)^4 = C \cdot R^2 \cdot 16D^4 = 16T$. So doubling the diameter increases the thrust by a factor of 16.

7. Let $g(x) = \frac{1}{x-1}$.

- a) (3 points) Evaluate and simplify the following expression.

$$g(a+1) - \frac{1}{a}$$

$$g(a+1) - \frac{1}{a} = \frac{1}{(a+1)-1} - \frac{1}{a} = \frac{1}{a} - \frac{1}{a} = 0$$

- b) (3 points) Solve $g(x) = -2$.

$$\begin{aligned} 2\frac{1}{x-1} &= -2 \\ x-1 &= \frac{1}{-2} \\ x &= \frac{1}{2} \end{aligned}$$

- c) (3 points) Find the average rate of change of $g(x)$ over the interval $10 \leq x \leq 12$.

$$\frac{\frac{1}{12-1} - \frac{1}{10-1}}{12-10} = \frac{1}{99}$$

- d) (3 points) For what values of x is the graph of $g(x)$ concave down?

When graphing the function on the calculator, it is clear that the function is concave down for all real numbers less than 1.

- e) (3 points) What is the equation of the horizontal asymptote of $g(x)$?

$$y = 0.$$

- f) (3 points) What is the domain of $g(x)$?

The domain of f is all real numbers except 1.

- g) (3 points) What is the range of $g(x)$?

The range of f is all real numbers except 0.

8. The number of asthma sufferers in the world was about 84 million in 1990 and 130 million in 2001. Let N represent the number of asthma sufferers (in millions) worldwide t years after 1990.

a) (4 points) Write N as a linear function of t .

We compute the slope as $m = \frac{130 - 84}{11} = \frac{46}{11} \cong 4.182$. Therefore, $N = 84 + \frac{46}{11}t$.

b) (4 points) What does the slope of this linear function tell you about asthma sufferers?

The number of worldwide asthma sufferers is increasing by 4.182 million people each year.

c) (4 points) Of the asthma sufferers in the world, 9% live in the United States. Assuming this relationship will still hold, how many American asthmatics will there be in 2004?

Our model predicts that $84 + \frac{46}{11} \cdot 14 \cong 106.55$ million people worldwide will have asthma. 9% of this is approximately 9.6 million people.

9. The table below gives the approximate number of cell phone subscribers, S , in millions, worldwide.

Year	1995	1996	1997	1998	1999	2000
Subscribers	91	138	210	320	485	738

- a) (4 points) Demonstrate using the data in the table why it makes sense to model this data with an exponential function.

When we compute growth factors for the intervals in the table, we see that they are $138/91 \cong 1.516$, $210/138 \cong 1.52$, $320/210 \cong 1.524$, $485/320 \cong 1.516$, and $738/485 \cong 1.522$. Since they are all close to 1.52, it makes sense to model this data with an exponential function.

- b) (4 points) Write a formula for S as an exponential function of t , the number of years since 1995.

$$S = 91 \cdot (1.52)^t.$$

- c) (4 points) In a single sentence describe how the number of cell phone subscribers has been changing since 1995. Use everyday words and do not use the symbols S or t .

The number of cell phone subscribers has been increasing by 52% each year.

10. Let $D(p)$ represent the number of iced cappuccinos sold each week by a coffeehouse when the price is set at p cents each.

a) (3 points) What does the equation $D(225) = 180$ tell you in everyday terms?

The coffeehouse sells 180 iced cappuccinos a week when they are priced at \$2.25 each.

b) (3 points) What does the expression $D^{-1}(200)$ represent?

It is a price at which the coffeehouse sells 200 iced cappuccinos a week.

c) (3 points) The coffeehouse sells n iced cappuccinos when they charge the average price in their area, t cents. Thus, $D(t) = n$. Write an expression which represents the number of iced cappuccinos that the coffeehouse would sell if they charged one and a half times the average price.

$$D(1.5t)$$