

MATH 105 — SECOND UNIFORM EXAM

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MICHIGAN

March 26, 2003, 6 pm - 7:30 pm

NAME: _____

ID NUMBER: _____

SIGNATURE: _____

INSTRUCTOR: _____

SECTION NO: _____

General Instructions: Do not open this exam until you are told to begin. The exam consists of 12 questions on 9 pages (including this cover). The exam has a 100 point scale.

Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.

Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. If you use graphs or tables to obtain an answer, be certain to provide an explanation or a sketch of the graph to make it clear how you arrived at your solution.

Use units where appropriate.

You may use your calculator, but no other outside materials.

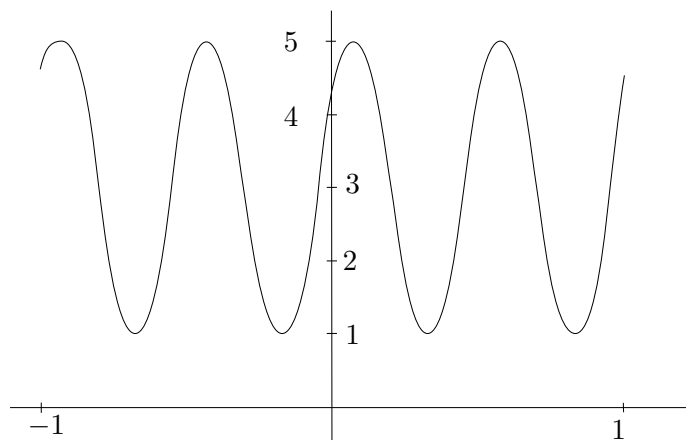
PROBLEM	POINTS	SCORE
1	3	
2	4	
3	5	
4	9	
5	12	
6	5	
7	3	
8	5	
9	9	
10	11	
11	21	
12	13	
TOTAL	100	

1. (3 points) A town has a population of 3000 people at year $t = 0$. Write a formula for the population, P , of the town t years later if the town shrinks at a *continuous rate* of 2% per year.

2. (4 points) Find all solutions of the equation $3 \cos(x/2) = 0.9$ in the interval $0 \leq x \leq \pi$.

3. (5 points) Which choice gives the amplitude and period of the graph of the periodic function in the figure below?

	Amplitude	Period
(a)	2	2
(b)	2	3
(c)	2	1/2
(d)	4	2
(e)	4	1/2



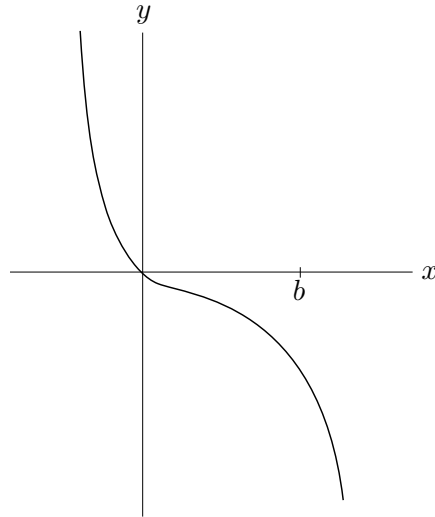
The correct choice is _____.

4. (9 points) In the figure below, the value b is labelled on the x -axis. On the y -axis, locate and label the following output values as accurately as possible.

a) $f(b)$

b) $-2f(b)$

c) $f\left(\frac{1}{2}b\right)$



5. (12 points) The U.S. population in millions is $P(t)$ today where t is in years. Match each statement (I)-(IV) with one of the formulas (a)-(h).

- | | |
|--|-------|
| I. The population 10 years before today. | _____ |
| II. Today's population plus 10 million immigrants. | _____ |
| III. Ten percent of the population we have today. | _____ |
| IV. The population after 100,000 people have moved out of the country. | _____ |

(a) $P(t) - 10$

(d) $P(t - 10)$

(g) $0.1P(t)$

(b) $P(t) + 10$

(e) $P(t + 10)$

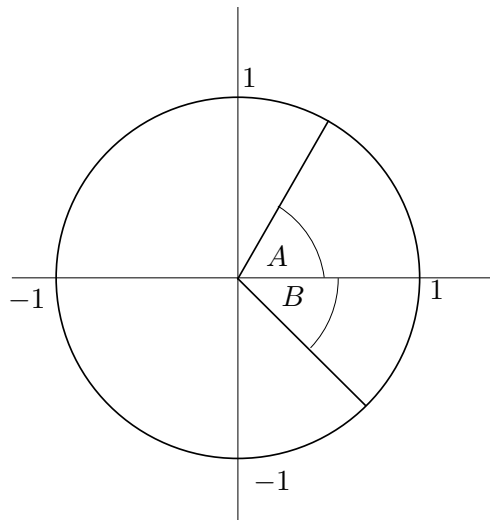
(c) $P(t) + 0.1$

(f) $P(t) - 0.1$

(h) $\frac{P(t)}{0.1}$

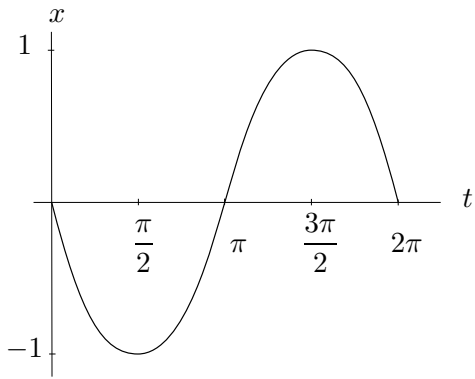
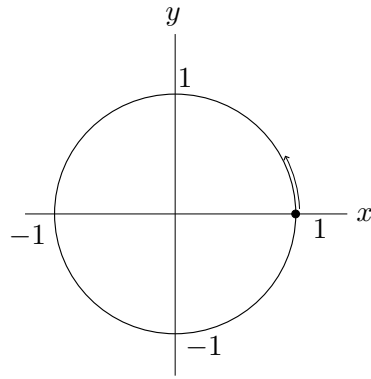
6. (5 points) Which of the following is the set of approximate values for the sine and cosine of angles A and B in the figure below? *Circle your answer.*

- (a) $\sin A \approx 0.5$, $\cos A \approx 0.85$, $\sin B \approx -0.7$, $\cos B \approx 0.7$.
- (b) $\sin A \approx 0.85$, $\cos A \approx 0.5$, $\sin B \approx -0.7$, $\cos B \approx 0.7$.
- (c) $\sin A \approx 0.5$, $\cos A \approx 0.85$, $\sin B \approx 0.7$, $\cos B \approx 0.7$.
- (d) $\sin A \approx 0.85$, $\cos A \approx 0.5$, $\sin B \approx 0.7$, $\cos B \approx 0.7$.

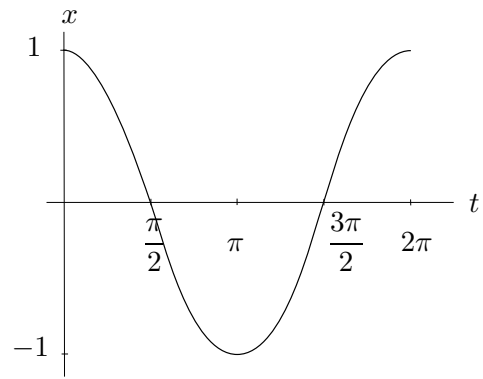


7. (3 points) Suppose the number of rabbits on Easter Island is growing exponentially in time. If it takes 3 years for the population to go from 502 rabbits to 1004 rabbits, how long does it take for the population to go from 1004 rabbits to 2008 rabbits?

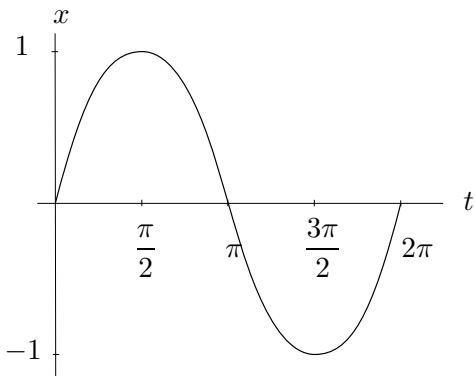
8. (5 points) Consider a point on the unit circle starting at an angle of zero and rotating counterclockwise at a constant rate as shown below. Which of the graphs represents the x -coordinate of this point as a function of time? *Circle your answer.*



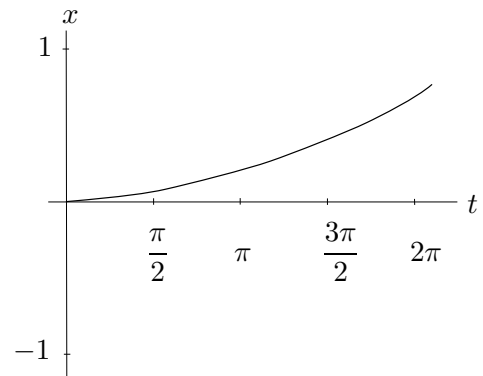
(A)



(B)

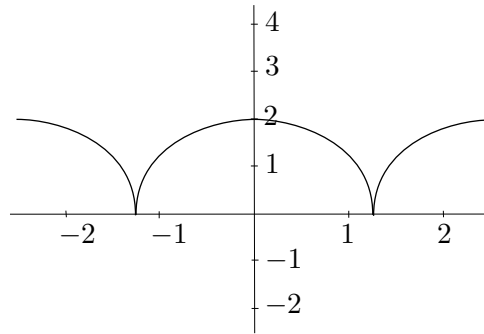


(C)



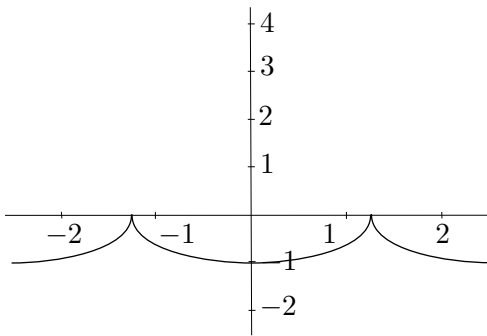
(D)

9. For this problem, a portion of the graph of $y = f(x)$ is given in the figure below.

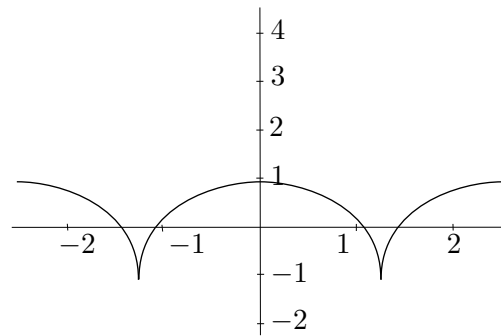


Use the graphs (A)-(D) below to answer the following questions. Each question may have more than one correct response: give all correct responses. Here, c , h and k are constants.

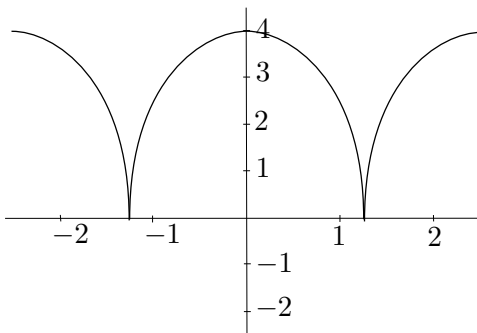
- a) (3 points) Which of the below could be the graph of $y = cf(x)$? _____
- b) (3 points) Which of the below could be the graph of $y = f(x) - k$? _____
- c) (3 points) Which of the below could be the graph of $y = f(x - h)$? _____



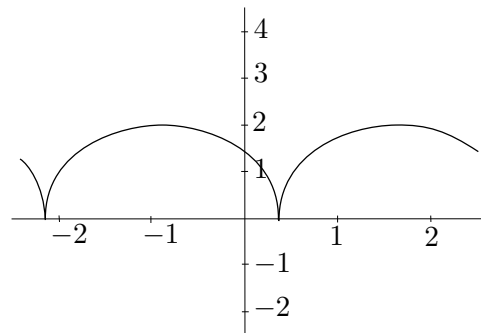
(A)



(B)



(C)



(D)

10. Suppose that $N(p) = -\frac{1}{10}(p - 65)^2 + 360$ represents the profit (in dollars) a vendor earns each day by selling lemonade on the Diag for p cents per cup.

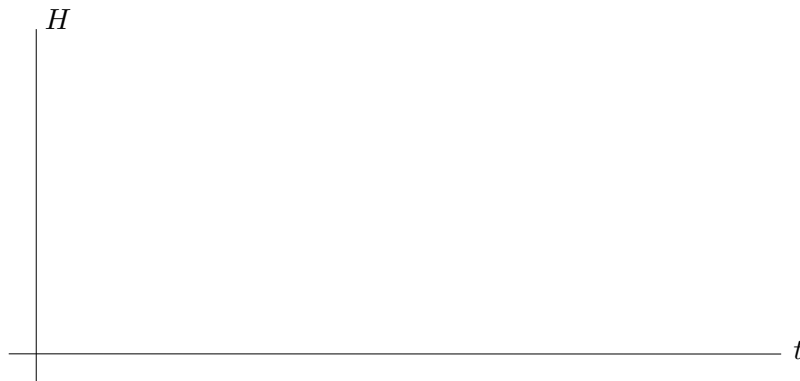
- a) (2 points) State the coordinates of the vertex of $N(p)$.
- b) (2 points) What price should the vendor charge for a cup of lemonade in order to earn the most profit per day?
- c) (2 points) What is the most profit this vendor can earn each day?
- d) (5 points) Solve $N(p) = 0$. Then, in one complete sentence, describe what your answers mean in terms of *profits* and the *price of lemonade*.

11. Let $H = f(t)$ represent a student's daily math study time t weeks after the start of each term. On the first night of the term ($t = 0$), the student's math study time was at a low of 0.5 hours. Exactly three weeks later, the student's math study time reached its first high of 6 hours. In this problem, we assume that the student's math study time varied sinusoidally (i.e. as a sine or cosine function) over the course of the 15 week term.

a) (3 points) State the equation of the midline of $H = f(t)$. Then, in one complete sentence, interpret this quantity in practical terms.

b) (2 points) What is the period of $H = f(t)$? Remember to include units in your answer.

c) (6 points) Sketch a graph of $H = f(t)$ for $0 \leq t \leq 15$. Be sure to label any important points.

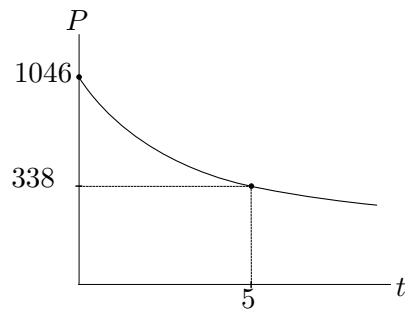


d) (3 points) Assuming that, like many students, this student studied most on the evening before each exam, determine how many days into the term she took the second exam.

e) (4 points) Find a formula for $H = f(t)$.

f) (3 points) Exactly four weeks after the start of the term, this student studied at the library from 6pm to midnight. She split her time between math and history. How long did she study history?

12. A treaty to protect the ozone layer has produced dramatic declines in global production of chlorofluorocarbons (CFCs). The figure below shows a graph of production of CFCs (in thousand ODP tons¹) as a function of the number of years since 1989.



a) (6 points) Find an exponential formula for the global CFC production, P , (in thousand ODP tons) as a function of the number of years, t , since the beginning of 1989.

b) (3 points) What was the annual percent decay rate, according to this formula?

c) (4 points) During what year was the production of CFC's equal to 200 thousand ODP tons?

¹An *ODP ton* is a measurement of the *ozone depleting potential* of the contaminant.