

MATH 105 — FINAL EXAM

DEPARTMENT OF MATHEMATICS
University of Michigan

April 21, 2003, 8 am –10 am

NAME: _____

ID NUMBER: _____

SIGNATURE: _____

INSTRUCTOR: _____

SECTION NO: _____

General Instructions: Do not open this exam until you are told to begin. The exam consists of 12 questions on 9 pages (including this cover). The exam has a 100 point scale.

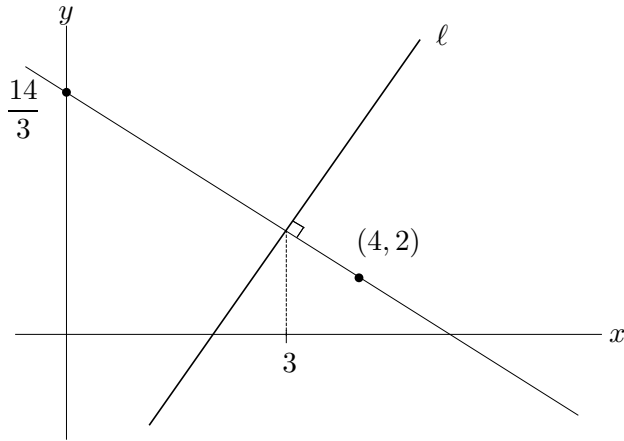
Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.

Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. Unless explicitly stated, no credit will be given for answers that do not show how they are derived. If you use graphs or tables to obtain an answer, be certain to provide an explanation (and a sketch of the graph, if that is the method) to make it clear how you arrived at your solution. Use units where appropriate.

You may use your calculator, but no other outside materials.

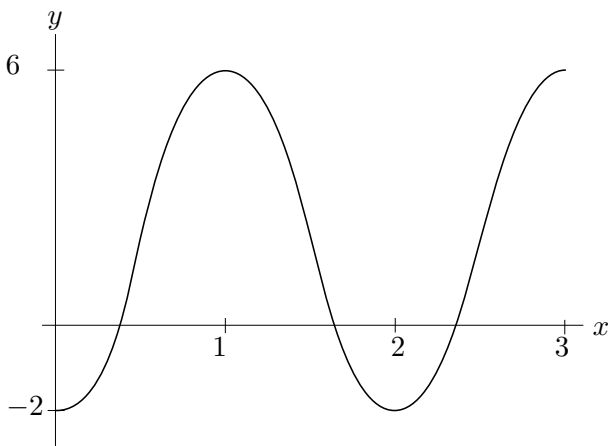
PROBLEM	POINTS	SCORE
1	4	
2	4	
3	4	
4	16	
5	8	
6	9	
7	18	
8	8	
9	4	
10	8	
11	12	
12	5	
TOTAL	100	

1. (4 points) Find a possible formula for the line ℓ .



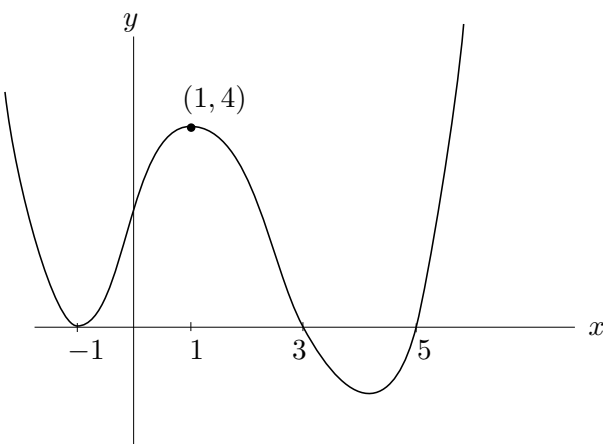
$$y = \frac{3}{2}x - \frac{11}{6}$$

2. (4 points) Find a possible formula for the following trigonometric function.



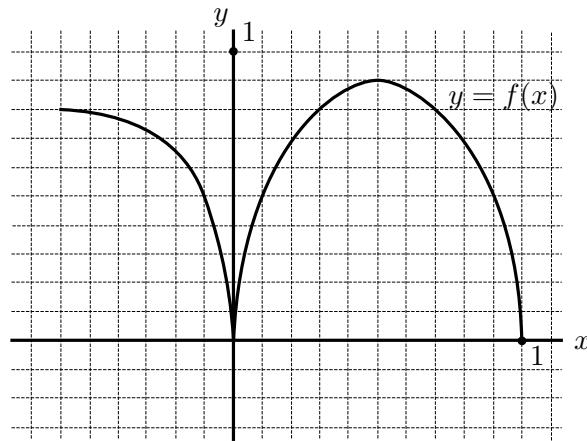
$$y = 2 - 4\cos(\pi x)$$

3. (4 points) Find a possible formula for the polynomial function graphed below.



$$y = \frac{1}{4}(x + 1)^2(x - 3)(x - 5)$$

4. The complete graph of a function $f(x)$ is given below. Use this graph to answer the questions which follow.



(a) (2 points) What is the domain of f ?

$$-0.6 \leq x \leq 1$$

(b) (2 points) What is the range of f ?

$$0 \leq y \leq 0.9$$

(c) (2 points) On what interval(s) is the function increasing?

$$0 \leq x \leq 0.5$$

(d) (2 points) On what interval(s) is the function concave down?

$$-0.6 \leq x \leq 1$$

(e) (2 points) Does f have an inverse?

No, it takes the value 0.5 at three different places.

(f) (3 points) Find the average rate of change of f over the interval $0.3 \leq x \leq 0.7$.

The average rate of change over this interval is zero.

(g) (3 points) Find all approximate values of x , if any, for which $f(x) = 0.5$.

$$-0.1, \quad 0.1, \quad 0.9$$

5. (8 points) This question concerns the tables given below.

x	$f(x)$
10	112
15	98
20	84
25	70
30	56

x	$g(x)$
-2	16
-1	24
0	36
1	54
2	81

x	$h(x)$
-3	-3
-2	0
-1	1
0	0
1	-3

One of the tables of data comes from a linear function, one from an exponential function and one from a quadratic function. Identify which is which and write a formula for each.

The function $f(x) = -14x + 252$ is linear. The function $g(x) = 36(1.5)^x$ is exponential. The function $h(x) = -x(x - 2)$ is quadratic.

6. Parts (a) and (b) of this question are separate.

(a) (4 points) Suppose that $p(x)$ is a cubic polynomial whose leading coefficient is -3 and whose constant term is 6 . Then

$$p(0) = \underline{\quad \mathbf{6} \quad},$$

and as $x \rightarrow \infty$,

$$p(x) \rightarrow \underline{\quad -\infty \quad}.$$

(b) (5 points) The information below describes a rational function g . Find a possible formula for this function.

The graph of $y = g(x)$ has two vertical asymptotes: one at $x = -2$ and one at $x = 1$. It has a horizontal asymptote of $y = 3$. The graph of g crosses the x -axis twice: once at $x = 4$, and once at $x = 5$.

$$g(x) = \mathbf{6} \frac{(x - 4)(x - 5)}{(x - 1)(x + 2)}$$

7. Parts (a), (b) and (c) of this question are separate.

(a) (12 points) For an experiment in your biology class, you begin to monitor the size P of a population of bacteria in thousands of organisms. Your measurements are modelled by the function $P = f(t) = 37.8(1.044)^t$, where t is in minutes.

1. Describe the population of bacteria in words.

It grows at a constant percentage rate of 4.4% per minute from a size of 37.8 thousand organisms at the start of the experiment.

2. Evaluate $f(50)$. What does this quantity tell you about the bacteria?

$f(50) \approx 325.5$. This is the number of bacteria, in thousands, 50 minutes after the start of the experiment.

3. Find a formula for $f^{-1}(P)$ in terms of P .

$$f^{-1}(P) = \frac{1}{\ln(1.044)} \ln\left(\frac{P}{37.8}\right)$$

4. Evaluate $f^{-1}(50)$. What does this quantity tell you about the bacteria?

$f^{-1}(50) \approx 6.5$. This means it takes about 6.5 minutes for the bacteria population to reach 50 thousand.

(b) (4 points) The sales, S , in thousands of units, of a seasonal product are modelled by

$$S = 58.3 - 32.5 \cos \frac{\pi t}{6}$$

where t is the time in months and $t = 0$ corresponds to June 1st.

1. What is the maximum number of units sold?

90.8 thousand

2. Would you expect the product to be air conditioners or snowboards? Explain why.

Since the sales are at a minimum on June 1st and a maximum on December 1st, it is more likely that the product is snowboards.

(c) (2 points) Consider a plane flying on a direct route between two cities. The distance s (in miles) travelled in t hours is $s = 560t$. What information about the flight does the slope of this line provide?

It is the speed of the plane in miles per hour.

8. (4 points) Parts (a) and (b) of this question are separate.

- (a) The length L of a steel rod (in centimeters) varies with its temperature H in degrees Fahrenheit. This relationship is described by a function $L = f(H)$. The rod's temperature at a time t hours after noon is given by the function $H = g(t)$. What is the practical meaning of $f(g(2)) = 3$?

This means that the steel rod is 3 centimeters long at 2pm.

- (b) (4 points) Consider the functions $v(x) = \frac{1+x}{e^{-x}}$ and $w(x) = \ln x$. Write a formula for the composition $u(x) = w(v(x))$. Simplify completely.

$$\mathbf{u(x) = \ln\left(\frac{1+x}{e^{-x}}\right) = \ln(1+x) + x}$$

9. (4 points) This question has two parts.

- (a) What is the definition of a rational function?

A rational function is the quotient $\frac{p(x)}{q(x)}$ of two polynomials, $p(x)$ and $q(x)$.

- (b) If $m(x) = x^{-2}$ and $n(x) = 2x^3$, demonstrate that $m(x) + n(x)$ is a rational function by putting it in the correct algebraic form.

$$\mathbf{m(x) + n(x) = \frac{2x^5 + 1}{x^2}}$$

10. Police have used the formula $s = \sqrt{30Cd}$ to estimate the speed s (in mph) at which a car was travelling if it skidded d feet. The parameter C is the coefficient of friction determined by the kind of road (concrete, asphalt, gravel, tar) and whether the road was wet or dry. The following are some values of C .

	Concrete	Tar
Wet	0.4	0.5
Dry	0.8	1.0

- (a) (4 points) Write a function which describes the relationship between s and d on a dry tar road. A car leaves a skid mark of about 141 feet. How fast was it going?

The proper functional relationship is

$$s = \sqrt{30d}.$$

The car's speed was approximately 65 miles per hour.

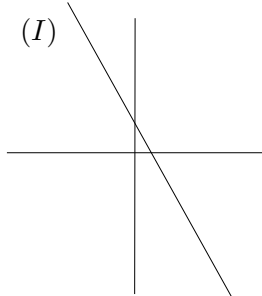
- (b) (4 points) Write a function which describes the relationship between s and d on a wet concrete road. At 55mph, about how many feet will a car skid?

The proper functional relationship is

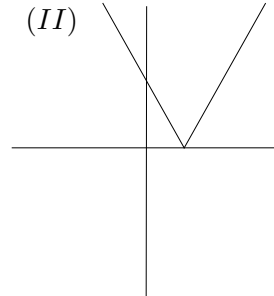
$$s = \sqrt{12d}.$$

The car skidded approximately 252 feet.

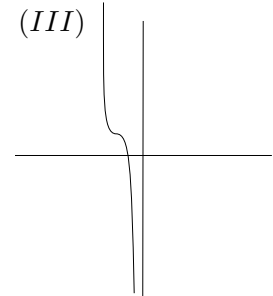
11. (12 points) For each of the graphs below, select the formula beneath the graph which *best fits* the behavior of the graph. In each case, assume that A , B and C are positive real numbers. (Circle your choice.) **Answers are in bold.**



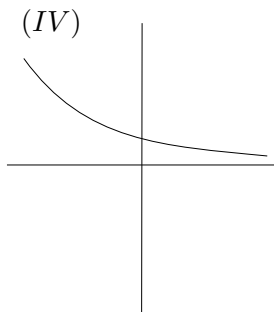
- (a) $y = Ax + B$
 (b) $y = -Ax - B$
 (c) $y = \mathbf{B - Ax}$
 (d) $y = (x + A)/(x + A)$



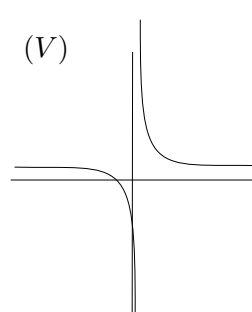
- (a) $y = |\mathbf{x - A}|$
 (b) $y = |x + A|$
 (c) $y = |x| - A$
 (d) $y = |x| + A$



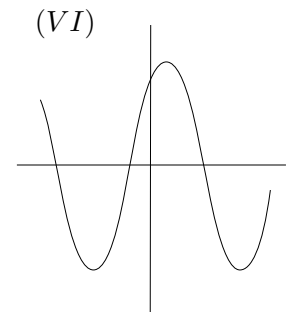
- (a) $y = -Ax^5 + B$
 (b) $y = Ax^3 + B$
 (c) $y = -\mathbf{A(x + B)^5 + C}$
 (d) $y = A(x + B)^5 + C$



- (a) $y = -\ln(x + A)$
 (b) $y = \mathbf{(1/A)^x}$
 (c) $y = -A^x$
 (d) $y = -e^x$



- (a) $y = A(x - B)/(x + C)$
 (b) $y = -A(x - B)/(x + C)$
 (c) $y = \mathbf{A(x + B)/(x - C)}$
 (d) $y = -A(x + B)/(x - C)$



- (a) $y = \mathbf{A \sin(x + B)}$
 (b) $y = -\cos x$
 (c) $y = -A \sin x + B$
 (d) $y = A \sin x - B$

12. (5 points) Explain how it is possible that a function which does not have an inverse can be considered as one that does have an inverse when the variables are given a physical meaning. Use a specific example to illustrate such a situation.

One way that this is possible is that the physical situation restricts the domain of the function under consideration. With this added restriction, the function becomes one-one and thus has an inverse. For example, the function $A = \pi r^2$ does not have an inverse, because both $r = 1$ and $r = -1$ give the same output of $A = \pi$. But if we use this function to model the area A of a circle given its radius r , things change. Since radius is a distance, it can never be negative. So we must restrict the domain from all real numbers to all real numbers $r \geq 0$. Now the function becomes invertible. Graphically, we are ignoring the left-hand side of the parabola like in the picture below.

