• **Due Date:** November 25 or 26, 2019 (Your instructor will tell you the exact date and time.)

• It is important that you try these problems **before your first meeting** with your team. Remember, the first meeting with your team is for the members of the team to discuss the arguments used by each member of the team to find their solutions. It is also a time to address any questions that each member of the team may have about the assignment.

• The team homework roles are

  – **Manager:** organizes and runs the meetings.
  – **Reporter:** writes a record of the meeting, including the roles of every member, how often the team met, how the assignment went, and any challenges and successes the team encountered. This reporter sheet is attached as the top sheet of your final draft.
  – **Scribe:** writes up a single final draft of the homework assignment to turn in.
  – **Clarifier:** makes sure everyone is on the same page and understands the problem’s statements and solutions.

Assign roles to each team member before beginning the team homework and rotate these roles between team members for every assignment. If your team has only three members, combine the roles of manager and clarifier into one; if it has five members, there should be two clarifiers.

• Please refer to [https://instruct.math.lsa.umich.edu/support/teamhomework/](https://instruct.math.lsa.umich.edu/support/teamhomework/) for full details on writing team homeworks.

• Highlighted text will be used to add commentary to the problems. In most cases this can be safely skipped, but you are encouraged to read it to gain context for both the problem and, hopefully, the course as a whole.

Please complete this checklist before submitting your team homework.

- □ Our solution includes a **restatement of the problem** in our own words, the way we understood it.
  It would be clear to someone who has not seen the question what the set up is and what we will do in our solution. In particular, all functions and variables have been defined, all graphs, formulas, and tables have been reproduced.

- □ Our solution is written in **full sentences**.
  In particular, all our computations are part of sentences, and every step is clear.

- □ Our solution includes **all the steps** we took to arrive at the answer.
  In particular, all the calculations, graphs, tables we used are included.

- □ Every step of our solution is **thoroughly explained** and **justified**.
  Another Math 115 student, who did not understand how to solve this question before, would understand it after reading our solution.
1. Custom cubics

Here’s a simple example to demonstrate how you might go about finding all the possible values of constants that satisfy a statement, or, in this example, multiple statements.

**Problem:** Consider the quadratic function \( f(x) = Bx^2 + Cx + D \) where \( B, C, \) and \( D \) are constants. Find all possible values of \( B, C, \) and \( D \) such that

- \( f(x) \) is an even function,
- the graph of \( f(x) \) is always concave down, and
- \( f(x) \) passes through the point \((1, 3)\).

**Solution:** First, for \( f(x) \) to be an even function, we need \( f(x) = f(-x) \), which means

\[
Bx^2 +Cx +D = Bx^2 -Cx +D.
\]

Since this means \( C = -C \), we must have \( C = 0 \). Second, for the graph of \( f(x) \) to be concave down, we need \( f''(x) = 2B \) to be negative, so \( B < 0 \). Finally, for \( f(x) \) to pass through the point \((1, 3)\), we need \( B(1)^2 + C(1) + D = 3 \), and we already know that \( C = 0 \), so we need \( B + D = 3 \), giving \( D = 3 - B \).

**Final Answer:** The values of \( B, C, \) and \( D \) that satisfy the conditions above are all the values such that \( C = 0, B < 0, \) and \( D = 3 - B \). This is the best way to describe all possible solutions. We need \( C = 0 \) and \( B < 0 \), and once we choose \( B \), the third statement tells us how to find \( D \). For example, if \( B = -2 \) then \( D \) must be equal to \( 3 - (-2) = 5 \).

Your answers below might similarly include equalities, inequalities, the words “and” or “or”, etc. There might be one, a few, infinitely many, or even no possible values of the constants that satisfy a given statement.

Recall that a cubic polynomial is a function of the form

\[
f(x) = Ax^3 + Bx^2 + Cx + D
\]

for constants \( A, B, C, \) and \( D \). For each of the following families of functions, either describe all possible values of \( A, B, C, \) and \( D \) such that \( f(x) \) will satisfy the prescribed conditions, or explain why the conditions are impossible to meet.

(a) The family of cubic polynomials that have a critical point at \((0, 1)\).

(b) The family of cubic polynomials that have an inflection point at \((0, 0)\) and a local maximum at \(x = 2\). Make sure to show enough work to justify that there really is an inflection point at \(x = 0\), and that there really is a local maximum at \(x = 2\).

(c) The family of cubic polynomials that are odd functions, that pass through the point \((1, 2)\), and such that the tangent line at \((1, 2)\) is given by the formula \( y = 2 - 3(x - 1) \).

(d) The family of cubic polynomials that are odd functions, that pass through the point \((1, 1)\), and such that the quadratic approximation at \(x = 1\) is given by the formula \( y = 1 + 2(x - 1) + 2(x - 1)^2 \).
2. It’s just a weather balloon!

(a) Alan and Sadie are monitoring a pair of unusually shaped weather balloons at Area 51. One balloon is shaped like a sphere of radius \( r \) feet, and the other balloon is shaped like a cube with side length \( x \) feet. As they watch, the volumes of the balloons appear to fluctuate (due to either inflation or deflation, as the case may be).

i. Write a formula which expresses the combined volume \( V \) of the two balloons in terms of the radius \( r \) and the side length \( x \).

ii. At a particular instant, the first balloon has \( r = 3 \), the second balloon has \( x = 5 \), and the combined volume of the balloons is increasing at a rate of 60 cubic feet per minute.

A. If, at this instant, \( \frac{dx}{dt} = 0.50 \), find \( \frac{dr}{dt} \). Then explain in context what this would mean about the weather balloons.

B. Instead, suppose that at this instant \( \frac{dr}{dt} = 0 \). Find \( \frac{dx}{dt} \), and explain in context what this would mean about the weather balloons.

C. At this instant, would it be possible for \( \frac{dr}{dt} < 0 \)? If so, find possible values of \( \frac{dx}{dt} \) and \( \frac{dr}{dt} \) and explain in context what this would mean about the weather balloons. If it is not possible, explain in context why not.

(b) Unbeknownst to Alan and Sadie, a mysterious grey van started following them after they arrived back in town from their trip to Area 51. As they drove back to their headquarters along Fourth Street, the van approached along a perpendicular road, Alien Avenue.

Let \( a \) be the distance in miles between Alan and Sadie’s car and the intersection of Alien Avenue and Fourth Street. Let \( r \) be the distance in miles between the van and the intersection. Let \( d \) be the distance in miles between their car and the mysterious van.

i. The CIA operatives in the van are trained to maintain a constant distance of exactly \( d = 17 \) miles from their targets. At the moment when \( a = 15 \), Alan and Sadie are driving at a speed of 45 miles per hour. How fast do the CIA operatives in the van have to drive at that moment in order to maintain a constant distance \( d = 17 \) from the car?

ii. At this same instant, what is the rate of change of the area, in square miles, of the triangle formed by the points at Alan and Sadie’s car, the CIA van, and the street intersection? Is the area increasing or decreasing?
3. Working for NASA

Business is booming at Atlantic SlopCorp. The Nevada regional office, called Nevada Atlantic SlopCorp & Associates (NASA) is negotiating the sale of a large quantity of rare radioactive pig slop to its best customer, a top secret government facility called Area 51.

(a) After long negotiations, NASA has agreed to use special pricing system with Area 51, which offers a discount on large purchase orders. Let $q$ be the quantity of slop, in thousands of gallons, that NASA produces.

- If $q \leq 10$ thousand gallons, then Area 51 will purchase all of the slop at a negotiated price of $11 - 0.1q$ million dollars per thousand gallons.
- If $q > 10$, then the first 10 thousand gallons will be sold to Area 51 at a price of 10 million dollars per thousand gallons, and the excess slop will be illegally sold off to various casino owners in Las Vegas at a black market price of 3.5 million dollars per thousand gallons.

Give a piecewise formula for NASA’s revenue $R(q)$, in millions of dollars, from producing $q$ thousand gallons of slop. **Make sure that your formula is continuous at $q = 10$.**

(b) The cost, in millions of dollars, to produce $q$ thousands of gallons of radioactive slop is given by the function $C(q)$.

$$C(q) = \frac{3}{400}q^3 - \frac{9}{20}q^2 + 12q + 10$$

i. What is the fixed cost of NASA’s slop manufacturing operation?

ii. At which values of $q$ in the interval $0 \leq q \leq 40$ does $MR(q) = MC(q)$? You may use a calculator on this part. Your answers do not need to be in exact form.

iii. Assume that NASA can produce at most 40 thousand gallons of slop. Determine the quantity $q$ of slop, in thousands of gallons, that NASA should produce in order to maximize their profit. **Make sure you have considered all values of $q$ at which the maximum profit could occur.**