• **Due Date:** January 31 or February 1, 2019 (Your instructor will tell you the exact date and time.)

• It is important that you try these problems **before your first meeting** with your team. Remember, the first meeting with your team is for the members of the team to discuss the arguments used by each member of the team to find their solutions. It is also a time to address any questions that each member of the team may have about the assignment.

• The team homework roles are
  
  – **Manager:** organizes and runs the meetings.
  
  – **Reporter:** writes a record of the meeting, including the roles of every member, how often the team met, how the assignment went, and any challenges and successes the team encountered. This reporter sheet is attached as the top sheet of your final draft.
  
  – **Scribe:** writes up a single final draft of the homework assignment to turn in.
  
  – **Clarifier:** makes sure everyone is on the same page and understands the problem’s statements and solutions.

Assign roles to each team member before beginning the team homework and rotate these roles between team members for every assignment. If your team has only three members, combine the roles of manager and clarifier into one.

• Please refer to [https://instruct.math.lsa.umich.edu/support/teamhomework/](https://instruct.math.lsa.umich.edu/support/teamhomework/) for full details on writing team homeworks.

• Highlighted text will be used to add commentary to the problems. In most cases this can be safely skipped, but you are encouraged to read it to gain context for both the problem, and hopefully the course as a whole.

Please complete this checklist before submitting your team homework.

☐ Our solution includes a **restatement of the problem** in our own words, the way we understood it.

  It would be clear to someone who has not seen the question what the set up is and what we will do in our solution. In particular, all functions and variables have been defined, all graphs, formulas, and tables have been reproduced.

☐ Our solution is written in **full sentences**.

  In particular, all our computations are part of sentences, and every step is clear.

☐ Our solution includes **all the steps** we took to arrive at the answer.

  In particular, all the calculations, graphs, tables we used are included.

☐ Every step of our solution is **thoroughly explained** and **justified**.

  Another Math 115 student, who did not understand how to solve this question before, would understand it after reading our solution.
1. Happy Cycling

The primary purpose of science is to model nature, and mathematics serves as an indispensable tool in this pursuit. We observe natural processes all around us and then try to describe them mathematically using functions, with the main goal of predicting their behavior in the future. It doesn’t take long to realize that a defining feature of many natural phenomena is periodicity, defined as the tendency to recur at intervals. The sun rises and sets each day, and in that same span of time the tides flow in and out approximately twice. Seasons beckon in a kaleidoscope of weather patterns in a yearly cycle, and hummingbirds flap their wings 70 times in a second, about the same rate at which an internal combustion engine completes a 4-stroke cycle. Trigonometric functions are great for modeling these phenomena since they also have this key property of periodicity. In this problem we’re going to work with a particular physical situation, see how we can model it with trigonometric functions, and then answer predictive questions using our model.

The following table lists daily average high and low temperatures, in degrees Fahrenheit, on the first day of each month for Ann Arbor.

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>Aug</th>
<th>Sept</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Highs</strong></td>
<td>31</td>
<td>34</td>
<td>46</td>
<td>60</td>
<td>71</td>
<td>80</td>
<td>84</td>
<td>82</td>
<td>75</td>
<td>62</td>
<td>48</td>
<td>36</td>
</tr>
<tr>
<td><strong>Lows</strong></td>
<td>17</td>
<td>19</td>
<td>27</td>
<td>38</td>
<td>49</td>
<td>58</td>
<td>62</td>
<td>61</td>
<td>54</td>
<td>43</td>
<td>33</td>
<td>23</td>
</tr>
</tbody>
</table>

(a) While this table is useful, it’s often nicer to have a continuous function when making predictions. Write a sinusoidal function \( H(m) \) that models the daily highs in Fahrenheit, \( m \) months after August 1st. Your function should have the coldest daily high occurring on January 1st and the warmest daily high occurring on July 1st, and these values should match those in the table. Write down a table of values for \( H(m) \) for \( m = 0 \) up to 11. Carefully graph \( H(m) \).

(b) Notice that your function \( H(m) \) does not perfectly fit the recorded data in the original table. For which of the data points above is \( H(m) \) least accurate?

(c) Write a sinusoidal function \( L(m) \) that models the daily lows in degrees Fahrenheit, \( m \) months after August 1st. As in part (a), your function should have the coldest daily low occurring on January 1st and the warmest daily low occurring on July 1st, and these values should match those in the table. Graph \( L(m) \) on the same axes as \( H(m) \). Can you obtain \( L(m) \) from \( H(m) \) using a series of function transformations? If so, write down a list of transformations you could perform. If not, explain why not.

(d) Now, let’s use one of your models to make a useful real world prediction. Suppose the University of Michigan decides to turn on air conditioning in all of their buildings whenever the daily high temperature exceeds 65 degrees Fahrenheit. According to your modeling function \( H(m) \), when during the period \( m = 0 \) to \( m = 12 \) do you expect that the university will use air conditioning? (This roughly corresponds to an academic year.) On approximately what dates do you expect the university to turn the air conditioning on and off?

This kind of prediction happens all the time in order to make budgeting decisions and meet yearly energy consumption standards.
2. Split your Infinities

The following questions concern the function

\[
k(t) = \frac{3t^4}{(t + 2)(t - 2)^3}.
\]

It may be useful to graph \(k(t)\) using Desmos (http://www.desmos.com).

(a) Find all horizontal and vertical asymptotes of \(k(t)\).

(b) Consider the function \(m(t) = (t^2 - 4) \cdot k(t)\). Find all horizontal and vertical asymptotes of \(m(t)\).

(c) For each of the following, find a function that meets the given criteria, or explain why it is impossible to find such a function. It may be helpful to remember that constant functions are a simple kind of polynomial, and that polynomials are a simple kind of rational function.

i. a polynomial \(p(t)\) so that \(f(t) = p(t) \cdot k(t)\) has no horizontal asymptotes

ii. a polynomial \(p(t)\) so that \(g(t) = p(t) \cdot k(t)\) does not pass through the origin

iii. a rational function \(r(t)\) so that \(h(t) = r(t) \cdot k(t)\) has the same vertical asymptotes as \(k(t)\), but has a horizontal asymptote at \(y = -4\)

iv. a rational function \(r(t)\) so that \(j(t) = r(t) \cdot k(t)\) has the same horizontal asymptote as \(k(t)\), all the vertical asymptotes of \(k(t)\), and an additional vertical asymptote at \(t = 4\)

v. a rational function \(r(t)\) so that \(\lim_{t \to a} r(t) \cdot k(t)\) exists for all real numbers \(a\)

(d) The function \(s(t) = \frac{3t^4}{(t + 2)^2(t - 2)^2}\) has the same horizontal and vertical asymptotes as \(k(t)\).

How do these two functions differ? In particular, you should be sure to compare the behavior of these functions near their vertical asymptotes, and to show how the differences in their formulas explain this behavior.
3. Push it to the Limit

Consider the following functions. Note that the graph of \( c(x) \) is constant on the interval \([3, 5]\):

\[
f(x) = \begin{cases} 
3x^2 + 2x + 2 & x < 2 \\
\frac{3x^2 + 2x + 2}{(x+1)(x-5)} & x = 2 \\
x^2 - 4x + 10 & x > 2 
\end{cases}
\]

(a) Carefully graph \( f(x) \). Pay close attention to continuity, and the behavior of \( f(x) \) as it approaches any vertical asymptotes from the left and right.

(b) Find the following limits using the graphs above:

\[
\lim_{x \to 4} a(x) \quad \lim_{x \to 4} b(x) \quad \lim_{x \to 4} c(x)
\]

Explain in words what happens to the values of \( a(x) \), \( b(x) \), and \( c(x) \) as \( x \) approaches 4 from the left and right. Particularly, describe how these situations are qualitatively different.

(c) Calculate each of the following limits or explain why it does not exist. It may be helpful to create a table of values approximating the limit, but this is not necessary.

i. \( \lim_{x \to 4} f(c(x)) \)

ii. \( \lim_{x \to 4} f(a(x)) \)

iii. \( \lim_{x \to 4} f(b(x)) \)

iv. \( \lim_{x \to 4} f(a(x)) \), assuming \( a(x) \) is an odd function

v. \( \lim_{z \to 0} f \left( \frac{1}{z} + 2z \right) \)  \( \text{Hint: Be sure to consider both the left- and right-hand limits} \)