• **Due Date:** February 28 or March 1, 2019 (Your instructor will tell you the exact date and time.)

• It is important that you try these problems **before your first meeting** with your team. Remember, the first meeting with your team is for the members of the team to discuss the arguments used by each member of the team to find their solutions. It is also a time to address any questions that each member of the team may have about the assignment.

• The team homework roles are
  
  – **Manager:** organizes and runs the meetings.
  – **Reporter:** writes a record of the meeting, including the roles of every member, how often the team met, how the assignment went, and any challenges and successes the team encountered. This reporter sheet is attached as the top sheet of your final draft.
  – **Scribe:** writes up a single final draft of the homework assignment to turn in.
  – **Clarifier:** makes sure everyone is on the same page and understands the problem’s statements and solutions.

Assign roles to each team member before beginning the team homework and rotate these roles between team members for every assignment. If your team has only three members, combine the roles of manager and clarifier into one.

• Please refer to [https://instruct.math.lsa.umich.edu/support/teamhomework/](https://instruct.math.lsa.umich.edu/support/teamhomework/) for full details on writing team homeworks.

• Highlighted text will be used to add commentary to the problems. In most cases this can be safely skipped, but you are encouraged to read it to gain context for both the problem, and hopefully the course as a whole.

Please complete this checklist before submitting your team homework.

- [ ] Our solution includes a **restatement of the problem** in our own words, the way we understood it.
  
  It would be clear to someone who has not seen the question what the set up is and what we will do in our solution. In particular, all functions and variables have been defined, all graphs, formulas, and tables have been reproduced.

- [ ] Our solution is written in **full sentences**.
  
  In particular, all our computations are part of sentences, and every step is clear.

- [ ] Our solution includes **all the steps** we took to arrive at the answer.
  
  In particular, all the calculations, graphs, tables we used are included.

- [ ] Every step of our solution is **thoroughly explained** and **justified**.
  
  Another Math 115 student, who did not understand how to solve this question before, would understand it after reading our solution.
1. **Down on the Corner**

Below is the graph of a function $f(x)$.

Some additional information about $f(x)$:

- On the interval $[-2, 0]$, the function is given by $f(x) = -x^2$.
- On the interval $[0, 2]$, the function is given by $f(x) = x^2$.
- $f(x)$ is linear on all other intervals.

(a) For what values of $x$ is $f(x)$ not continuous?

(b) For what values of $x$ is $f(x)$ not differentiable?

(c) Draw a well-labelled graph of $f'(x)$. Indicate clearly where it is discontinuous or undefined.

(d) For what values of $x$ is $f'(x)$ not differentiable?

(e) Draw a well-labelled graph of $f''(x)$. Indicate clearly where it is discontinuous or undefined.

(f) Let $g(x) = e^x(f(x))^2$. Calculate $g'(1)$ exactly.
2. Down on the Farm

As farming practices improve, farmers are able to produce more of a given crop per acre of farmland. A prime example of this is provided by soybeans, for which the yield per acre has been steadily increasing for the past 30 years. Below are some values of the function $Y(t)$, which gives the average yield, measured in bushels per acre, $t$ years after 1988. We also provide some values for the derivative $Y'(t)$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>4</th>
<th>10</th>
<th>16</th>
<th>22</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y(t)$</td>
<td>26</td>
<td>37</td>
<td>38</td>
<td>42</td>
<td>44</td>
<td>42</td>
<td>52</td>
</tr>
<tr>
<td>$Y'(t)$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>1.5</td>
<td>-0.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

The price of soybeans tends to fluctuate from year to year. Let $P(t)$ be the price of one bushel of soybeans, in dollars, $t$ years after 1988. Some values of $P(t)$ and its derivative are shown below.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>4</th>
<th>10</th>
<th>16</th>
<th>22</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(t)$</td>
<td>7.9</td>
<td>5.8</td>
<td>6.6</td>
<td>8.4</td>
<td>9.5</td>
<td>12</td>
<td>9.6</td>
</tr>
<tr>
<td>$P'(t)$</td>
<td>-0.5</td>
<td>0.2</td>
<td>0.8</td>
<td>0.5</td>
<td>-0.1</td>
<td>-0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

(a) Consider each of the following functions.

i. The annual revenue from one acre of soybeans is given by $A(t) = P(t)Y(t)$. Calculate $A'(4)$. What are the units of $A(t)$?

ii. Let $M(x) = P(x/12)$ give the price of soybeans $x$ months after 1988. Calculate $M'(192)$.

iii. Let $S(t) = \dfrac{Y(t)}{\cos(2\pi t) + 10}$ be the seasonally adjusted yield. Calculate $S'(24)$.

(b) Estimate each of the following values.

i. $P''(7)$

ii. $\dfrac{d}{dt} \left( P'(t)Y(t) \right)$ at $t = 30$

iii. $\dfrac{d}{dt} \left( e^{Y'(t)} \right)$ at $t = 0$
3. Constants are Changing

Here’s a simple example to demonstrate how you might go about finding all the possible values of constants that satisfy a statement, or, in this example, multiple statements.

**Problem:** Consider the linear function \( f(x) = mx + b \) where \( m \) and \( b \) are constants. Find all possible values of \( m \) and \( b \) such that

- the \( y \)-intercept of \( f(x) \) is greater than zero, and
- \( f(x) \) passes through the point \((2, 8)\).

**Solution:** First, in order to have a positive \( y \)-intercept, we need \( b > 0 \). Second, in order to pass through \((2, 8)\), we should have \( 8 = 2m + b \), so \( m = \frac{8-b}{2} \).

**Final Answer:** The values of \( m \) and \( b \) that satisfy the conditions above are all the values such that \( b > 0 \) and \( m = \frac{8-b}{2} \).

This is the best way to describe all possible solutions. We need \( b > 0 \), and once we choose \( b \), the second statement tells us how to find \( m \). For example, if \( b = 2 \) then \( m \) must be equal to \( \frac{8-2}{2} = 3 \).

Your answers below might similarly include equalities, inequalities, the words “and” or “or”, etc. There might be one, a few, infinitely many, or even no possible values of the constants that satisfy a given statement.

Consider the function \( h(x) \) where \( k \) and \( B \) are constants:

\[
h(x) = \begin{cases} 
  x^2 - Bx + 1 & x \leq k \\
  -6x & x > k.
\end{cases}
\]

Note that each part below is a separate problem.

(a) Find all possible values of \( B \) and \( k \) such that \( h(2) = 0 \).

(b) Find all possible values of \( B \) and \( k \) such that \( h(0) = 2 \).

(c) Find all possible values of \( B \) and \( k \) such that \( h'(1) = 5 \).

(d) Find all possible values of \( B \) and \( k \) such that \( h(x) \) is decreasing at \( x = 0 \).

(e) Find all possible values of \( B \) and \( k \) such that the function \( h(x) \) is continuous.

(f) Find all possible values of \( B \) and \( k \) such that the function \( h(x) \) is differentiable.