• **Due Date:** March 21 or 22, 2019 (Your instructor will tell you the exact date and time.)

• It is important that you try these problems **before your first meeting** with your team. Remember, the first meeting with your team is for the members of the team to discuss the arguments used by each member of the team to find their solutions. It is also a time to address any questions that each member of the team may have about the assignment.

• The team homework roles are
  
  – **Manager:** organizes and runs the meetings.
  
  – **Reporter:** writes a record of the meeting, including the roles of every member, how often the team met, how the assignment went, and any challenges and successes the team encountered. This reporter sheet is attached as the top sheet of your final draft.
  
  – **Scribe:** writes up a single final draft of the homework assignment to turn in.
  
  – **Clarifier:** makes sure everyone is on the same page and understands the problem’s statements and solutions.

Assign roles to each team member before beginning the team homework and rotate these roles between team members for every assignment. If your team has only three members, combine the roles of manager and clarifier into one.

• Please refer to [https://instruct.math.lsa.umich.edu/support/teamhomework/](https://instruct.math.lsa.umich.edu/support/teamhomework/) for full details on writing team homeworks.

• Highlighted text will be used to add commentary to the problems. In most cases this can be safely skipped, but you are encouraged to read it to gain context for both the problem, and hopefully the course as a whole.

Please complete this checklist before submitting your team homework.

- Our solution includes a restatement of the problem in our own words, the way we understood it.
  - It would be clear to someone who has not seen the question what the set up is and what we will do in our solution. In particular, all functions and variables have been defined, all graphs, formulas, and tables have been reproduced.

- Our solution is written in full sentences.
  - In particular, all our computations are part of sentences, and every step is clear.

- Our solution includes all the steps we took to arrive at the answer.
  - In particular, all the calculations, graphs, tables we used are included.

- Every step of our solution is thoroughly explained and justified.
  - Another Math 115 student, who did not understand how to solve this question before, would understand it after reading our solution.
1. Surfer Rosa

The jury is out on how long we'll have to wait for fully autonomous vehicles to hit the roads. However, many companies have already had success developing autonomous robots in more restricted applications such as autonomous vacuums, lawn mowers, and theme park shuttles. In order to operate most effectively, these robots need to be able to calculate optimal behavior (this is called “path planning”) in a variety of circumstances.

Suppose you’ve developed an autonomous vehicle called ABC (Autonomous Boat Car) that can rescue drowning victims when deployed at a beach.

(a) When ABC spots a distressed swimmer off the coast, it must calculate the fastest way to reach them. It will do this by driving along the beach for some distance and then turning and travelling diagonally through the water. ABC can travel 20 mph along the beach, and 10 mph through water.

(i) ABC spots a distressed surfer named Rosa 3 miles up the coast and 1 mile out at sea. How far should ABC travel up the coast before it enters the water?

(ii) How long does it take ABC to reach Rosa?

(iii) Suppose you update ABC’s hardware so that it can now travel 20 mph through water (and still can travel 20 mph on the beach). In the same situation, how far should it travel up the coast before it enters the water? Explain why your answer makes sense.

(b) You are redesigning the hull of ABC to be an isosceles triangle with area 20 m$^2$ as shown below at right. The cost of the siding to make the base of the triangle is 100 dollars per meter. The siding for the other two sides must be made out of heavy duty material, so has a higher cost of 300 dollars per meter. The floor making up the inside of the triangle costs 20 dollars per square meter.

(i) Write a formula for $b$ in terms of $h$.

(ii) Express the cost of the hull $C(h)$ as a function of $h$.

(iii) In the context of the problem, what is the domain of $C(h)$? Explain.
2. The Trooper

(a) An important component of military planning is assessing an enemy weapon’s range. This allows military leadership to place strategic assets outside of the weapon’s range. For example, a Russian general in the Crimean war would have been very interested in the range of Britain’s new (at the time) 68-pounder gun.

The theoretical range of a projectile is the maximum distance it can travel before it hits the ground. The theoretical range depends both on its initial velocity $v$ (known as muzzle velocity for guns and cannons) as well as the angle $\theta$ it makes from the ground. The equation is:

$$ R(\theta) = \frac{v^2}{9.8} \sin(2\theta) $$

i. The British 68-pounder had a muzzle velocity of 480 m/s. What was its theoretical range?

ii. In reality, the 68-pounder was only ever fired at angles between 0 and $\pi/12$ radians. What was its theoretical range in this case? What firing angle would give this theoretical range?

(b) A military testing facility wants to analyze the effectiveness of a new artillery gun at varying distances. They come up with a scoring system that incorporates accuracy, precision, mobility, and other factors. Several months of testing reveals that the effectiveness $E(d)$ as a function of distance $d$ in thousands of meters is given by the formula

$$ E(d) = -100(-d^2 + d - 1)e^{-d} $$

i. In the context of the problem, what should the domain of $E(d)$ be?

ii. What value of $d$ maximizes $E(d)$?

iii. Realistically, the artillery gun must be placed at least 500 meters away from the target. Based on this new information, what value of $d$ will maximize $E(d)$?

iv. In an upcoming battle it will be impossible to place the artillery gun closer than 1500 meters from its target. How far away should it be placed to maximize effectiveness?
3. Mean Street

Below is the graph of $g'(x)$, the derivative of $g(x)$. The function $g(x)$ is continuous on the interval $[-5, 5]$. Also graphed is the function

$$f(x) = \begin{cases} \ln(-x) & x < 0 \\ -x & x > 0. \end{cases}$$

Note that $f(x)$ is not defined for $x = 0$.

(a) Find the critical points of $f(x)$.
(b) Find the critical points of $g(x)$.
(c) Using calculus, classify each critical point of $g(x)$ as a local minimum, local maximum, or neither.
(d) Find the inflection points of $g(x)$.
(e) Consider a differentiable function $h(x)$ with $h'(x) > 0$. Find the critical points of $h(g(x))$.
(f) Find the critical points of $(f(x))^2$.
(g) For each of the following intervals, does $f(x)$ satisfy the hypotheses of the Mean Value Theorem? Does $f(x)$ satisfy the conclusion of the Mean Value Theorem? It is not necessary to find any $c$ values (where $c$ is the point in the conclusion of the theorem), but be sure to provide solid reasoning.

i. $[-1, 1]$
ii. $[-5, 1]$
iii. $[5, 10]$