• **Due Date:** April 18 or 19, 2019 (Your instructor will tell you the exact date and time.)

• It is important that you try these problems **before your first meeting** with your team. Remember, the first meeting with your team is for the members of the team to discuss the arguments used by each member of the team to find their solutions. It is also a time to address any questions that each member of the team may have about the assignment.

• The team homework roles are
  
  – **Manager:** organizes and runs the meetings.
  
  – **Reporter:** writes a record of the meeting, including the roles of every member, how often the team met, how the assignment went, and any challenges and successes the team encountered. This reporter sheet is attached as the top sheet of your final draft.
  
  – **Scribe:** writes up a single final draft of the homework assignment to turn in.
  
  – **Clarifier:** makes sure everyone is on the same page and understands the problem’s statements and solutions.

Assign roles to each team member before beginning the team homework and rotate these roles between team members for every assignment. If your team has only three members, combine the roles of manager and clarifier into one.

• Please refer to [https://instruct.math.lsa.umich.edu/support/teamhomework/](https://instruct.math.lsa.umich.edu/support/teamhomework/) for full details on writing team homeworks.

• Highlighted text will be used to add commentary to the problems. In most cases this can be safely skipped, but you are encouraged to read it to gain context for both the problem, and hopefully the course as a whole.

Please complete this checklist before submitting your team homework.

| □ Our solution includes a **restatement of the problem** in our own words, the way we understood it. |
| It would be clear to someone who has not seen the question what the set up is and what we will do in our solution. In particular, all functions and variables have been defined, all graphs, formulas, and tables have been reproduced. |

| □ Our solution is written in **full sentences**. |
| In particular, all our computations are part of sentences, and every step is clear. |

| □ Our solution includes **all the steps** we took to arrive at the answer. |
| In particular, all the calculations, graphs, tables we used are included. |

| □ Every step of our solution is **thoroughly explained** and **justified**. |
| Another Math 115 student, who did not understand how to solve this question before, would understand it after reading our solution. |
1. Problem 1

Wealth inequality in the United States is an increasingly heated issue that will likely play a significant role in the 2020 federal elections. We will examine a few statistics related to this subject in the following exercise.

(a) Let $M(t)$ be the annual income of an average US household in the middle 20% of incomes, in thousands of dollars per year, $t$ years after 1979. Let $U(t)$ be the annual income of an average US household in the top 5% of incomes, in thousands dollars per year, $t$ years after 1979. The following table provides some values of $M(t)$ and $U(t)$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>10</th>
<th>16</th>
<th>21</th>
<th>28</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(t)$</td>
<td>57</td>
<td>60</td>
<td>60</td>
<td>66</td>
<td>67</td>
<td>62</td>
</tr>
<tr>
<td>$U(t)$</td>
<td>197</td>
<td>260</td>
<td>300</td>
<td>363</td>
<td>343</td>
<td>323</td>
</tr>
</tbody>
</table>

i. Write an expression that represents the total income earned from 1979 to 2010 by an average household in the middle 20%. Estimate the value of this integral using both left and a right Riemann sums. Your sums should use all the given data, so note that the width of each subinterval may not be constant.

ii. Write an expression for the average yearly income, over the years 1995 to 2007, of an average US household in the top 5% of incomes. Estimate this quantity using the table.

iii. Write an expression for the difference in the total income earned, from 1989 to 2007, between an average household in the middle 20% and an average household in the top 5%. Estimate this quantity using the table.

(b) Let $W(t)$ be the real value of the minimum wage\(^2\), in dollars, $t$ years after 1979. Provide a practical interpretation of each of the following mathematical statements.

i. $W(34) = 7.25$

ii. $W'(34) = -0.15$

iii. $\int_{30}^{34} W'(t) \, dt = 0.62$

iv. $\int_{2}^{0} \frac{W(t)}{2} \, dt = 8.41$

(c) Consider $W(t)$ as given in part (b). The following facts are true about $W(t)$. First, $W(0) = 8.67$, and second,

$$\int_{0}^{y} W'(t) \, dt = -0.26y \quad \text{for} \quad 0 \leq y \leq 10.$$

i. Find $W(3)$ and $W(7)$.

ii. What can you conclude about the real value of the minimum wage from 1979 to 1989? In particular, how was it changing during that time period?

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\(^1\)The given values are pretax income, inflation-adjusted to 2011 dollars, and rounded to simplify calculations. Data from [http://stateofworkingamerica.org/chart/swa-income-table-2-1-average-family-income/](http://stateofworkingamerica.org/chart/swa-income-table-2-1-average-family-income/).

\(^2\)This is the value of the minimum wage adjusted for inflation, given in 2013 dollars. Any numbers used here are based on [http://stateofworkingamerica.org/chart/swa-wages-figure-4-ae-real-minimum-wage/](http://stateofworkingamerica.org/chart/swa-wages-figure-4-ae-real-minimum-wage/).
2. Problem 2

Let $g(x)$ be a differentiable function defined on all real numbers. The graph of its derivative, $g'(x)$, is shown below. Also suppose that $g(2) = 3$.

Also define the differentiable, odd function $h(x)$ on all real numbers. Some values of $h(x)$ are given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x)$</td>
<td>0</td>
<td>-2</td>
<td>-1</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Calculate each of the following quantities or, if there isn’t enough information, explain why.

i. $\int_{-3}^{1} (g'(x) + 2) \, dx$

ii. $\int_{0}^{3} h(x) \, dx$

iii. $\int_{-4}^{2} (h'(x) + 2x) \, dx$

iv. $\int_{-2}^{2} 8h(x) \, dx$

v. $\int_{-5}^{8} g'(x + 4) \, dx$

(b) Carefully draw a well-labeled graph of $g(x)$.

(c) For each of the following functions, find an antiderivative. (In other words, find a function whose derivative is the function given.) Your answers may be in terms of $g(x)$, $h(x)$ or their derivatives.

i. $g'(x)h(x) + g(x)h'(x)$

ii. $g'(x)e^{g(x)}$

iii. $h'(2x + 7)$

iv. $g'(x) + 17 \cos(x)$
3. Problem 3

(a) Haley and Paige are delivery drivers for Father Jonathan’s Pizzeria (FJP’s). Their velocities in miles per hour, $t$ minutes after 8:00 p.m., are shown in the graph below. Haley’s velocity is given by the dashed line while Paige’s is the solid line. Positive velocity indicates movement away from the store while negative velocity indicates movement towards the store. At 8:00 p.m., Haley is at FJP’s, while Paige is 0.5 miles away.

In each of the following, your answers may be approximations.

i. At what time is Haley closest to FJP’s? How close is she?

ii. At what time is Haley furthest from FJP’s? How far away is she?

iii. At what time(s) are Haley and Paige the same distance from FJP’s?

iv. At what time(s) are Haley and Paige travelling at the same velocity?

v. At what time(s) are Haley and Paige travelling at the same speed?

(b) The pizzeria opens at 4:00 pm. Let $r(t)$ be the rate, in pizzas per hour, at which customers are ordering pizzas $t$ hours after opening. Initially, the pizzeria can cook 35 pizzas per hour. At 6 pm, another cook arrives, so the pizzeria can cook 50 pizzas per hour starting at that time.

Estimate the following quantities using the graph.

i. The number of pizzas that have been ordered but not yet cooked by 6:00 pm.

ii. The time(s) when the pizzeria is not backed up (i.e. when there are no pizza orders waiting to be cooked).

iii. The amount of time a customer should expect to wait if they order a pizza at 7 pm.