- **Due Date:** March 12 or 13, 2020 (Your instructor will tell you the exact date and time.)
- It is important that you try these problems **before your first meeting** with your team. Remember, the first meeting with your team is for the members of the team to discuss the arguments used by each member of the team to find their solutions. It is also a time to address any questions that each member of the team may have about the assignment.
- The team homework roles are
  - **Manager:** organizes and runs the meetings.
  - **Reporter:** writes a record of the meeting, including the roles of every member, how often the team met, how the assignment went, and any challenges and successes the team encountered. This reporter sheet is attached as the top sheet of your final draft.
  - **Scribe:** writes up a single final draft of the homework assignment to turn in.
  - **Clarifier:** makes sure everyone is on the same page and understands the problem’s statements and solutions.

Assign roles to each team member before beginning the team homework and rotate these roles between team members for every assignment. If your team has only three members, combine the roles of manager and clarifier into one; if it has five members, there should be two clarifiers.

- Please refer to [https://instruct.math.lsa.umich.edu/support/teamhomework/](https://instruct.math.lsa.umich.edu/support/teamhomework/) for full details on writing team homeworks.
- Highlighted text will be used to add commentary to the problems. In most cases this can be safely skipped, but you are encouraged to read it to gain context for both the problem and, hopefully, the course as a whole.

Please complete this checklist before submitting your team homework.

- [ ] Our solution includes a **restatement of the problem** in our own words, the way we understood it. It would be clear to someone who has not seen the question what the set up is and what we will do in our solution. In particular, all functions and variables have been defined, all graphs, formulas, and tables have been reproduced.

- [ ] Our solution is written in **full sentences**. In particular, all our computations are part of sentences, and every step is clear.

- [ ] Our solution includes **all the steps** we took to arrive at the answer. In particular, all the calculations, graphs, tables we used are included.

- [ ] Every step of our solution is **thoroughly explained** and **justified**. Another Math 115 student, who did not understand how to solve this question before, would understand it after reading our solution.
1. Consider the curve $C$ defined implicitly by the equation

$$x(y^2 - 4y) + 8y = 2y^2 - 3x + 5.$$ 

Throughout this problem, be sure to show every step of your algebraic work.

(a) Find all points on the curve $C$ that have a $x$-coordinate of 3.
(b) Find a formula for $dy/dx$. Be sure every step of your work is clear.
(c) Check that the point $(\frac{5}{3}, 4)$ lies on the curve $C$, and then find the equation of the tangent line to $C$ at this point.
(d) Find the coordinates of all points where $C$ has a horizontal tangent line, or explain why there are no such points.
(e) Find the coordinates of all points where $C$ has a vertical tangent line, or explain why there are no such points.
2. Isabelle is a bee keeper who wants to sell honey at the local farmers market. Let \( y = H(d) \) be the amount of honey, in pounds, that Isabelle will sell in a month if she charges \( d \) dollars per pound of honey. As Isabelle increases the price per pound, fewer people will buy her honey, so \( H(d) \) is a decreasing function. Some values of \( H(d) \) and \( H'(d) \) are given in the table below.

<table>
<thead>
<tr>
<th>( d )</th>
<th>5.25</th>
<th>6.50</th>
<th>7.40</th>
<th>8.10</th>
<th>9.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H(d) )</td>
<td>57</td>
<td>48</td>
<td>35</td>
<td>28</td>
<td>22</td>
</tr>
<tr>
<td>( H'(d) )</td>
<td>-7.4</td>
<td>-12.7</td>
<td>-10.2</td>
<td>-7.3</td>
<td>-5.1</td>
</tr>
</tbody>
</table>

Assume that between each pair of consecutive values of \( d \) given in the table, \( H'(d) \) is either always increasing or always decreasing.

(a) Find the linear approximation of \( H(d) \) at \( d = 8.10 \), and use it to estimate the amount of honey Isabelle will sell if she charges $8.00 per pound of honey. Is your estimate an overestimate or an underestimate? Explain.

(b) Isabelle’s friend Tom is from France and wants to sell her honey abroad. The amount of honey, in pounds, that Tom will sell if he charges \( k \) Euros per pound of honey is given by \( T(k) = 5\sqrt{H(1.08k)} \). Find the linear approximation of \( T(k) \) at \( k = 7.5 \), and use it to estimate the amount of honey Tom will sell if he charges 8 Euros per pound of honey.\(^1\)

(c) Explain why \( H(d) \) is an invertible function.

(d) Compute \( (H^{-1})'(48) \), and give a practical interpretation of your answer.

(e) Suppose Isabelle has exactly 50 pounds of honey. Find the linear approximation of \( H^{-1}(y) \) at \( y = 48 \), and use it to estimate the price Isabelle should charge if she wants to sell all of her honey.

(f) Let \( R(d) = d \cdot H(d) \) be Isabelle’s revenue in dollars when she charges \( d \) dollars per pound of honey. Compute \( R'(7.4) \) and give a practical interpretation of your answer.

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\(^1\)As of February 23, 2020, 1 Euro is equivalent to 1.08 dollars. In particular, 7.5 Euros is equivalent to 8.1 dollars.
3. If you deposit $a$ dollars into a bank account with an interest rate of $k\%$ per year, the value of your deposit after $t$ years is given by

$$a \left(1 + \frac{k}{100}\right)^t.$$ 

The Rule of 70 states that the time it takes for an investment like this to double is approximately 70 divided by the percent growth rate. For example, if the value of an investment is growing at a rate of 2% per year, then the Rule of 70 predicts that it will take approximately $\frac{70}{2} = 35$ years for the investment to double in value.

Your work through the first few parts of this problem will justify why the Rule of 70 works. Since the point of the rule is to be a simple approximation that can be done in your head, your work in this problem shouldn’t involve a calculator except perhaps to double-check some basic arithmetic (i.e. addition, subtraction, multiplication, or division).

(a) Show that the number of years it takes for your deposit to double in value is given by

$$\frac{\ln(2)}{\ln \left(1 + \frac{k}{100}\right)} \approx \frac{0.70}{\ln \left(1 + \frac{k}{100}\right)}.$$ 

(b) Compute the linear approximation $L(x)$ of $\ln(1 + x)$ at $x = 0$.

(c) Suppose that the interest rate is 8% per year. Use your linear approximation from (b) to estimate $\ln(1.08)$, the denominator of the formula from (a) when $k = 8$. Then, apply this estimate to the formula from (a) to approximate how long it takes for the deposit to double. Compare this calculation to the one described by the Rule of 70.

(d) Now use your linear approximation from (b) to estimate $\ln \left(1 + \frac{k}{100}\right)$ for any $k$ (i.e., without picking any value for $k$). How does this justify the Rule of 70?

(e) Compute the quadratic approximation $Q(x)$ of $\ln(1 + x)$ at $x = 0$.

(f) Using your formula from (a) and your quadratic approximation from (e), estimate the time it will take for your deposit to double in value if the interest rate is 8% per year.