• **Due Date:** around April 20, 2020 (Your instructor will tell you the exact date and time.)

• It is important that you try these problems **before your first meeting** with your team. Remember, the first meeting with your team is for the members of the team to discuss the arguments used by each member of the team to find their solutions. It is also a time to address any questions that each member of the team may have about the assignment.

• The team homework roles are
  
  – **Manager:** organizes and runs the meetings.
  – **Reporter:** writes a record of the meeting, including the roles of every member, how often the team met, how the assignment went, and any challenges and successes the team encountered. This reporter sheet is attached as the top sheet of your final draft.
  – **Scribe:** writes up a single final draft of the homework assignment to turn in.
  – **Clarifier:** makes sure everyone is on the same page and understands the problem’s statements and solutions.

Assign roles to each team member before beginning the team homework and rotate these roles between team members for every assignment. If your team has only three members, combine the roles of manager and clarifier into one; if it has five members, there should be two clarifiers.

• Please refer to [https://instruct.math.lsa.umich.edu/support/teamhomework/](https://instruct.math.lsa.umich.edu/support/teamhomework/) for full details on writing team homeworks.

• Highlighted text will be used to add commentary to the problems. In most cases this can be safely skipped, but you are encouraged to read it to gain context for both the problem and, hopefully, the course as a whole.

Please complete this checklist before submitting your team homework.

- Our solution includes a **restatement of the problem** in our own words, the way we understood it.
  It would be clear to someone who has not seen the question what the set up is and what we will do in our solution. In particular, all functions and variables have been defined, all graphs, formulas, and tables have been reproduced.

- Our solution is written in **full sentences**.
  In particular, all our computations are part of sentences, and every step is clear.

- Our solution includes **all the steps** we took to arrive at the answer.
  In particular, all the calculations, graphs, tables we used are included.

- Every step of our solution is **thoroughly explained** and **justified**.
  Another Math 115 student, who did not understand how to solve this question before, would understand it after reading our solution.
1. Terry is making a cake by pouring batter into a mold in the shape of a pyramid, as shown below to the left. In particular, the top opening of the mold is a rectangle with length 6 inches and width 8 inches, and the height of the mold is 12 inches. Also note that, when viewed from the side, the pyramid appears as one of the two triangles shown below to the right.

Let $B(t)$ be the rate, in cubic inches per second, at which Terry is pouring the batter into the mold $t$ seconds after he starts pouring. A table giving some values of $B(t)$ is shown below.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1.6</th>
<th>2.3</th>
<th>3.2</th>
<th>4.8</th>
<th>5.5</th>
<th>6.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(t)$</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>17</td>
<td>25</td>
<td>35</td>
<td>47</td>
</tr>
</tbody>
</table>

Recall that the volume of a pyramid with a rectangular base is given by $V = \frac{1}{3}Ah$, where $A$ is the area of the base and $h$ is the height of the pyramid.

(a) Write a formula in terms of $h$ for the volume of batter, in cubic inches, Terry has poured into the mold if the batter in the mold reaches a height of $h$ inches.

*Hint: You can find two pairs of similar triangles in the pyramid using the two triangles above to the right. One allows you to relate the height of the batter to the length of the rectangle forming the top surface of the batter. The other allows you to relate the height of the batter to the width of the rectangle forming the top surface of the batter.*

(b) Suppose that 2.3 seconds after Terry starts pouring the batter, the batter in the mold reaches a height of 2 inches.

i. How fast is the height of the batter in the mold rising at this time?

ii. How fast is the area of the top surface of the batter in the mold increasing 2.3 seconds after Terry starts pouring the batter?

(c) Assuming $B(t)$ is an increasing function, use a Riemann sum with four equal subdivisions to find an overestimate for the amount of batter, in cubic inches, that Terry pours into the mold during the first 6.4 seconds.
2. Zelda runs a factory that prints books.

(a) Let \( p(t) \) be the rate, in thousands of pages per hour, that a printing press in Zelda’s factory is printing pages \( t \) hours after 7 AM.

i. Give a practical interpretation of the integral

\[
\int_{1.5}^{8} p(t) \, dt.
\]

ii. Give a practical interpretation of the expression

\[
\frac{1}{3} \int_{4}^{7} p(t) \, dt.
\]

iii. Zelda doesn’t know when the printing press started running, but that it was sometime before 7 AM. She also knows that it printed 300,000 pages between the time it started running and 10 AM. Write an expression involving one or more integrals for the number of pages, in thousands, the printing press produced between the time it started running and 7 AM.

(b) The graph below shows the marginal revenue \( MR \) (dashed) and marginal cost \( MC \) (solid), in dollars per book, of printing \( q \) copies of a certain book.

![Graph showing MR and MC functions](image)

i. For what value(s) of \( q \) in the interval \([0, 20]\) is the cost function minimized?

ii. Let \( \pi(q) \) be Zelda’s profit from printing \( q \) copies of the book. What are the critical points of \( \pi(q) \) in the interval \([0, 20]\)?

iii. For what values of \( q \) in the interval \([0, 20]\) is the profit function \( \pi(q) \) maximized?

iv. For what values of \( q \) in the interval \([0, 20]\) is the profit function \( \pi(q) \) concave down? Express your answer as one or more intervals.
3. A portion of the graph of a function $k(x)$ is shown below. Note that the part of the graph on the interval $[-6, -4]$ can be obtained from the part of the graph on the interval $[-4, -2]$ by shifting it two units to the left and reflecting it over the $x$-axis. Also note that the part of the graph on the interval $(7, 9]$ is a semicircle.

Let $K(x)$ be the continuous antiderivative of $k(x)$ passing through the point $(0, -1)$. Sketch a large, detailed graph of $K(x)$, paying particularly closed attention to

- where $K(x)$ is and is not differentiable,
- the values of $K(x)$ at $x = -6, -2, 0, 3, 7,$ and $9$,
- where $K(x)$ is increasing/decreasing/constant,
- the concavity of $K(x)$. 

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