Determine whether the following integral converges or diverges. Show all of your work and indicate any theorems you used to conclude convergence or divergence of the integral.

\[ \int_0^1 \frac{x^{1/3} \cos(x) - 1/3 x^{-2/3} \sin(x)}{x^{2/3}} \, dx \]

(Hint: \( \frac{d}{dx} \sin(x) = x^{1/3} \cos(x) - 1/3 x^{-2/3} \sin(x) \).

\[ \int_0^1 \frac{x^{1/3} \cos(x) - 1/3 x^{-2/3} \sin(x)}{x^{2/3}} \, dx \]

\[ = \lim_{b \to 0^+} \int_b^1 \frac{x^{1/3} \cos(x) - 1/3 x^{-2/3} \sin(x)}{x^{2/3}} \, dx \]

\[ = \lim_{b \to 0^+} \left[ \sin(1) - \frac{\sin(b)}{b^{1/3}} \right] \]

\[ = \sin(1) - \lim_{b \to 0^+} \frac{\sin(b)}{b^{1/3}} \]

\[ = \sin(1) - \lim_{b \to 0^+} \frac{\cos(b)}{b^{1/3}} \]

\[ = \sin(1) - \frac{1}{3} b^{2/3} \cos(b) \]

\[ = \sin(1) - 0 \]

\[ = \sin(1) \]

Converges.

(Note that this problem did not ask for the value to which the integral converges. Some problems certainly could ask for that, in which case the value \( \sin(1) \) would be part of the final answer.)