Problem 1 : The function

\[ J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^n (n!)^2} \]

is called a Bessel function of order 0. This function first arose when Bessel solved Kepler’s equation for describing planetary motion. These functions have a wide range of applications, including the temperature distribution in a circular plate and the shape of a vibrating drumhead. Determine the domain of \( J_0(x) \) by finding the interval of convergence of the series defining \( J_0(x) \).

Problem 2 : Section 10.2, problem 30.

Problem 3 : Find the Taylor series of

\[ F(x) = \int_0^x \sin(t^4) \, dt \]

about \( x = 0 \).

Problem 4 : The electric potential, \( V \), at a distance \( d \) along the axis perpendicular to the center of a charged disc with radius \( r \) and constant charge density \( \rho \), is given by

\[ V = 2\pi \rho (\sqrt{d^2 + r^2} - d). \]

1. Show that for small \( d \),

\[ V \approx V_0 = 2\pi r \rho - 2\pi \rho d \]

2. Show that for large \( d \),

\[ V \approx V_\infty = \frac{\pi r^2 \rho}{d}. \]

Hint: Find an expansion of \( V \) in terms of powers of \( \frac{r}{d} \).

Graph \( V \), \( V_0 \) and \( V_\infty \) to visualize the validity of your expansions (use \( r = 1 \) and \( \rho = 1 \)).