In issue 2 of Derivative Girl, our hero has found the Calc1 forgetfulness device, but the code to turn it off involves the values of a function $f$, and its derivative. To turn off the device, she must enter all the unknown values in the following table:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t)$</td>
<td>9</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f'(t)$</td>
<td>2</td>
<td>$-9$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-3$</td>
<td></td>
</tr>
</tbody>
</table>

She knows $f$ is a differentiable function with continuous derivative, and some other facts about $f$:

- $\int_{-1}^{0} f'(f(t))f'(t) \, dt = -7$,
- $f(3) = f'(3)$,
- If $g(z) = z \int_{f(-1)}^{z} f'(t) \, dt$, then $g'(3) = 5$,
- $\int_{-1}^{3} tf''(t) \, dt = -2$, and
- $\int_{-1}^{2} f'(t) \, dt = 3$.

What are the missing values that she will calculate for the table?

2. Damian’s favorite hobby is convincing his friends that the area of a circle with radius $r$ is $\pi r^2$. He knows this area is given by $2 \int_{-r}^{r} \sqrt{r^2 - x^2} \, dx$. Unfortunately, none of the techniques he tries seems to help in integrating.

(a) His first thought is to try the substitution method with $w = r^2 - x^2$, but this doesn’t work. Do you agree? For the time being, he uses TRAP and MID. Find TRAP(4) and MID(3) when $r = 2$. Is each an overestimate or an underestimate?

(b) Using sigma notation, write formulas for RIGHT(n) and LEFT(n), again assuming $r = 2$ (for the definition of sigma notation, see page 283 of the text).

Damian’s older brother Richard remembers hearing that sometimes, instead of substituting the more complicated function $(r^2 - x^2)$ with an easier function $(w)$, it is helpful to replace the easy function $x$, with a more complicated function.

For the rest of this problem, let $r$ be a positive constant (that is, you should no longer assume $r = 2$).
(c) Let’s try this method. Start by letting \( x = r \sin(\theta) \). Now, this makes \( dx = r \cos(\theta)d\theta \). Use this replacement to rewrite the integral, including the bounds.

(d) Now, remembering that \( \cos^2(x) + \sin^2(x) = 1 \), simplify the function inside the integral, and finally evaluate it.

(e) Follow this method with the same replacement, \( x = r \sin(\theta) \) to evaluate \( \int_{\sqrt{2}/2}^{\sqrt{2}/2} \frac{4r}{\sqrt{r^2 - x^2}} \, dx \).

Remember \( \sin(\pi/4) = \frac{1}{\sqrt{2}} \) and \( \sin(-\pi/4) = -\frac{1}{\sqrt{2}} \).

Note: If you’d like to learn more about the setup to parts b) through d), check out the textbook §7.4, or look up trig substitution on Khan Academy. This method will not be tested nor required on any course exams (including the gateway), but it gives a way to solve integrals arising in many applications, including ones that show up in Chapter 8.