Some guidelines for your first assignment

- You must read and attempt the problems before meeting with your team. Even if you aren’t able to obtain all the answers, being prepared during the team meetings helps your group work more efficiently during the meeting.

- Don’t be discouraged if you cannot solve most of the problems on your own — this is perfectly normal. This is part of why you are being assigned to work on these assignments as a group; make sure to discuss your questions and ideas with your teammates.

- If your team is having trouble with a particular problem, try utilizing the Math Lab (our math tutoring center - see more details here: https://lsa.umich.edu/math/undergraduates/course-resources/math-lab.html) with your teammates to get help.

- Make sure everyone is involved and no-one feels excluded during the meetings. If you notice someone is shy, actively encourage them to contribute to the group!

- Ask your teammates to explain their reasoning behind their answers if you don’t understand it. Remember that all members of the team are responsible for this assignment, and everyone should be on board with what the team turns in.

- Write up your final solutions neatly, and make sure your explanations are clear and complete.
1. Maria receives an email from her coworker Brian, asking her to verify his work. She quickly realizes that he has made several incorrect statements, but she has more important things to work on right now. Can you explain to Brian why each of these statements is incorrect? For each one, you may give an example of a function $f(x)$ (either using a formula or a sketch) for which the statement is false.

In each statement, $f(x)$ is a continuous function on the interval $[0, 6]$ and $\text{LEFT}(n)$ and $\text{RIGHT}(n)$ are, respectively, the Left and Right estimates with $n$ subintervals for the definite integral $\int_0^6 f(x)dx$.

(a) $\text{RIGHT}(3)$ always gives a better estimate to $\int_0^6 f(x)dx$ than $\text{RIGHT}(2)$ does.
(b) For each value of $n$, $\text{LEFT}(n)$ and $\text{RIGHT}(n)$ must have different values.
(c) If $\text{RIGHT}(3)$ is an underestimate to $\int_0^6 f(x)dx$, then $\text{LEFT}(3)$ is an overestimate to $\int_0^6 f(x)dx$.
(d) If $\text{LEFT}(1)=\text{LEFT}(2)=\text{LEFT}(3)=\text{LEFT}(4)=\text{LEFT}(5)=\text{LEFT}(6)=0$, then $\int_0^6 f(x)dx = 0$. 
2. Vegetable Crossing is the new video game craze with a huge following. After releasing a new update, the producers track how the number of active players fluctuate. If \( t \) is the number of weeks since the update was released, then \( g(t) \), measured in millions of active players per week, gives the rate of change of the total number of active players of Vegetable Crossing. A graph of \( g(t) \) is given below. The function \( g(t) \) is piecewise linear, except on the interval \([0, 1]\) where it has the formula \( g(t) = 4t - 2t^2 \).

(a) A major component of the game involves playing the stork market, but the market crashes after a bug appears in the game, \( T \) weeks after the release of the update. For the first time after the release of the update, the total number of active players starts to decrease. What is the value of \( T \)?

(b) What is the average value of \( g(t) \) on the interval \([0, 5]\)? What does this mean in the context of this problem?

(c) Let \( G(t) \) be the total number of active players \( t \) weeks after the update, measured in millions, and assume that \( G(t) \) is continuous. In the 5 weeks after the update, the maximum number of active players was 12 million. Given that information, fill out the following table of values:

<table>
<thead>
<tr>
<th>( t )</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G(t) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(d) Sketch a graph of \( G(t) \) for \(-1 < t < 5\). Pay careful attention to where \( G(t) \) is increasing or decreasing, and concave up or concave down.
3. The function $H(x)$ is defined by

$$H(x) = \int_{3}^{f(x)} \sin(t^3) \, dt$$

where $f(x)$ is a differentiable function. Some values of $f(x)$ and $f'(x)$ are given in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$\frac{\pi}{2}$</th>
<th>0</th>
<th>$\left(\frac{\pi}{2}\right)^{1/3}$</th>
<th>3</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>$-\left(\frac{\pi}{2}\right)^{1/3}$</td>
<td>-3</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>$\pi$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>13</td>
<td>2</td>
<td>-5</td>
</tr>
</tbody>
</table>

(a) Without using a sketch of the graph of $\sin(x^3)$, explain why $\sin(x^3)$ is an odd function.
(b) Based on the information provided, for which values of $x$ must $H(x) = 0$?
(c) Find $H'(3)$. 