1. After the back-to-back epic losses, Ryan Rabbitt decides to cheer himself up with a delicious smoothie. He makes the smoothie with his fancy electric drink mixer. Mathematically, the container of the mixer has the shape of a paraboloid, which can be modeled as the surface obtained by rotating the region in the first quadrant bounded by the curves $y = 25$ and $y = x^2$ around the $y$-axis (all lengths are measured in centimeters). The diagrams below are intended to give you a sense of the situation but may not be drawn to scale.

(a) Ryan pours various kinds of liquid into the container all the way to the marked max line (the red dashed line in the diagram above), which is 18 centimeters above the bottom. Set up two integrals representing the total volume of the liquid inside the mixer: one with respect to $x$, and one with respect to $y$. Then find the total volume of liquid inside the mixer.

(b) Now Ryan goes on to mix his drink. The container spins around the $y$-axis at a very high speed, causing the liquid to move away from the center of the container. The result is the solid made by rotating the shaded (orange) region around the $y$-axis in the diagram on the next page. Note that this means that there is an empty space inside liquid that has the shape of a cylinder. Find the radius of that cylinder.
(c) Ryan dozes off before his drink is ready, and when he wakes up, the container has stopped spinning and the mixture has separated and become layered. Suppose the density of the layered liquid at height $y$ is $\delta(y)$ grams per cubic centimeters. Set up an integral that gives the total mass of the liquid inside the mixer.

2. Hannah Haire wants to retrace the course of the running race. She remembers the following information:

- Creating a grid with the Block M on the Diag as the origin and mapping the course as a set of points $(x, y)$ where $x$ is kilometers east of the Diag and $y$ is kilometers north of the Diag, the resulting curve passes the vertical line test, that is, $y = f(x)$ for some function $f$.
- The starting point of the course was two kilometers directly north of the Block M on the Diag.
- When she was $x$ kilometers east and $y$ kilometers north of the Diag, she had run a distance of

\[ s(x) = \int_0^x \sqrt{3w + 5} \, dw \]

kilometers.

(a) Use the information above to help Hannah to find a formula for $f(x)$.
(b) Approximate $s(10)$ using MID(2) and TRAP(4).