1. During the last week of classes before finals, Ryan Rabbitt starts to find coffee a daily necessity. Every morning, Ryan makes his pour-over coffee with a flat-bottom filter, which can be modeled as a cylinder with radius 5 centimeters and height 12 centimeters. He pours a liquid coffee mix into the filter, and the liquid slowly flows through the filter into a container below.

When filtering, the flow rate is proportional to the contact area between the liquid and the filter. (The “flow rate” is the rate at which the liquid is leaving the filter; the “contact area” is the area of the part of the coffee filter, including the bottom and part of the side, that is touching the liquid.)

(a) Let \( h \) be the height (in cm) of the surface of liquid above the bottom of the filter. Set up a differential equation of \( h \) with respect to the time \( t \) (in minutes) after filtration begins.

(Hint: Let \( V \) be the total volume of coffee mixture in the filter, and observe the chain rule

\[
\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt}.
\]

What is \( dV/dt \)? What is \( dV/dh \)?)

(b) Use separation of variables to find a formula for \( h(t) \). Your answer may involve unknown parameter(s).

(c) Suppose the filter is initially full when filtration begins, and the filtration process takes about 2.2 minutes, or more accurately,

\[
t = \frac{5}{4} \ln(29/5).
\]

Use this information to determine the parameter(s) for your formula for \( h(t) \) above.

(d) As fellow math students join his study group, Ryan realizes he needs a continuing source of coffee for the group members. To do so, Ryan keeps adding liquid to the filter at a constant rate of 6 cubic centimeters per minute. In the long run, what will be the height of the surface level of the coffee? (You should be able to answer this question without explicitly solving any differential equations!)
2. Four slope fields and four differential equations, each containing constants, are given below.

\[ \frac{dy}{dx} = (y - c)(x^2 - d) \]
\[ \frac{dy}{dx} = ay(b - y) \]
\[ \frac{dy}{dx} = px^2 + q \]
\[ \frac{dy}{dx} = x - y + k \]

(a) Pair the differential equations with the slope fields.

(b) For each differential equation: Do you have enough information to find the exact value of the constant(s)? If so, find them. If not, what can you say about their possible values? For example, can you say that one must be positive, or give a relationship between the two constants in an equation?

(c) For each slope field, find all equilibrium solutions and determine whether they are stable.