Math 116 – Team Homework Assignment #1, Winter 2020

• Due Date: January 23 or 24 (Your instructor will tell you the exact date and time.)
• Note: All problem, section, and page references are to the course textbook, which is the 7th edition of Calculus: Single Variable by Hughes-Hallett, Gleason, McCallum, et al.
• Consult the “Doing Team Homework” and “Team HW Tutorial” links on the course website for guidelines on doing Team Homework.
• Rotate roles and include a reporter’s page each week.
• Show complete work.

1. (a) i. Without using any antiderivative formulas or graphing, find an $x$-intercept for the graph of $y = H(x)$, where $H(x) = \int_{2}^{x} (2t - 1) \, dt$.

   ii. Use the Fundamental Theorem of Calculus (the version from 5.3) to rewrite the expression for $H(x)$ so that no integral sign appears. Use this to find all the $x$-intercepts of $y = H(x)$.

   iii. Use the previous part to find a value of $b \neq 2$ so that $J(x) = \int_{b}^{x} (2t - 1) \, dt$ is the same function as $H(x)$. Explain how you can tell $J(x) = H(x)$ for all $x$ without any computation.

(b) Recall that the entire family of antiderivatives of $f(x) = \frac{1}{1 + x^2}$ is given by $G(x) = \arctan(x) + C$, where $C$ can be any real number. By the Construction Theorem for Antiderivatives (the version of the Fundamental Theorem of Calculus from 6.4), for each real number $a$, the function given by $F(x) = \int_{a}^{x} \frac{1}{1 + t^2} \, dt$ is an antiderivative of $f(x) = \frac{1}{1 + x^2}$.

   i. Use the Fundamental Theorem of Calculus (the version from 5.3 again) to determine the value of $a$ so that $\int_{a}^{x} \frac{1}{1 + t^2} \, dt$ gives the particular antiderivative $K(x) = \arctan(x) - 1$.

   ii. Explain why it is impossible to find a real number $a$ such that $\int_{a}^{x} \frac{1}{1 + t^2} \, dt$ equals the function $M(x) = \arctan(x) - 2$. What can you conclude from this?

(c) i. Calculate $\frac{d}{dx} \int_{a}^{x} e^{\cos(t)} \, dt$. Use this derivative to explain why $P(x) = \int_{a}^{x} e^{\cos(t)} \, dt$ is an increasing function of $x$ on the interval $(-\infty, \infty)$, regardless of the value of the constant $a$.

   ii. For the function $G(x) = \int_{0}^{x^2} e^{\cos(t)} \, dt$, use a graph or two to determine which is larger, $G(-2)$ or $G(0)$ (feel free to use your calculator or Desmos to help graph $y = e^{\cos(t)}$). Does this contradict the previous part?

   iii. Calculate $\frac{d}{dx} \int_{0}^{x^2} e^{\cos(t)} \, dt$. 

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2. The finale song at a rock concert begins at 11:30pm. The table below shows the values of \( C(t) \), the rate of change of the volume level (measured in decibels/minute) during the finale at several times \( t \) (measured in minutes past 11:30 pm). Assume the finale song lasts for ten minutes.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C(t) )</td>
<td>10</td>
<td>10</td>
<td>−20</td>
<td>30</td>
<td>2</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

Suppose the rate of change of the volume is linear between every two consecutive times in the table.

(a) Sketch a clear, well-labeled graph of \( C(t) \) on the interval \([0, 10]\).

Three minutes after the start of the finale, the volume is 105 decibels. Let \( V(t) \) be the volume, in decibels, \( t \) minutes after 11:30pm.

(b) Using the previous part, sketch a clear, well-labeled graph of \( V(t) \) for \( t \) in the interval \([0, 10]\). Clearly indicate where \( V(t) \) is increasing, decreasing, concave up, and concave down.

(c) What was the average value of the rate of change of the volume during the finale?

(d) Let \( A(t) \) be the average volume during the first \( t \) minutes of the finale. Write an expression for \( A(t) \) using an integral and either \( C(t) \) or \( V(t) \).

(e) Calculate the rate of change of \( A(t) \) one minute after the finale has started. Compare this to the average value of the rate of change of the volume during the first minute of the finale.