1. (a) Consider a rocket blasting off from the earth’s surface in a video game. At $t = 10$ seconds after the rocket engines ignite, the rocket detaches from the platform and lifts off. Its speed after liftoff is given by $r(t) = \frac{300}{\sqrt{t(t+1)}}$ km/second, where $t$ is measured in seconds after the engines ignite (i.e. the domain of $r(t)$ is $[10, \infty)$). This speed function is accurate for all time after liftoff and the rocket is heading directly away from the earth’s surface.

   i. Assuming the video game runs forever, write an expression involving an integral that represents the altitude the rocket approaches as time goes on.
   ii. Find the altitude from part i. Give both an exact value and a decimal value, accurate to at least four digits. Be sure to show and use appropriate notation for your work and calculations. Hint: let $w = \sqrt{t}$.

(b) During an upcoming untethered space walk, an astronaut in a science fiction movie needs to position herself between 30 and 35 meters away from the space station she is currently aboard. The astronaut’s speed during the walk will be controlled by jets on her spacesuit that operate according to a pre-programmed speed function she cannot change once the walk begins. Like the rocket engines in the previous problem, the jets need 10 seconds to fire up after ignition. Let $s$ be the number of seconds after the jets first ignite. Note that the speed functions below are accurate for all time after the jets have fired up, i.e. that the domain of each function is $[10, \infty)$.

   i. Will a speed (in meters per second) given by $j_1(s) = 200 \frac{s^2}{s^2}$ position her within the correct distance of the station, or shoot her off into space? Use appropriate notation and calculations to support your answer.
   ii. After making the previous calculation, she decides that maybe she will instead try to program a speed (in meters per second) of $j_2(s) = 20 \frac{s}{s}$. How far from the space station would she travel with this program?
   iii. After determining these previous two speeds will not work, she considers a speed (in meters per second) of $j_3(s) = 20 \frac{\ln 10}{s \ln s}$. What would happen in this case?
   iv. Sketch a graph of the three speed functions from parts i-iii over the interval $[10, \infty)$. What do you notice?
2. Answer the following using the given table of values:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>π/2</th>
<th>2</th>
<th>3</th>
<th>7</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x)$</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>9</td>
<td>11</td>
<td>15</td>
<td>22</td>
<td>30</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>10</td>
<td>23</td>
<td>12</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

The function $f(x)$ is a positive, differentiable function, and its derivative $f'(x)$ is continuous and positive. Find the exact value of each of the following.

(a) $\int_0^{\pi/2} \cos(x)f'(\sin(x))e^{f(\sin(x))} \, dx$

(b) $\int_3^7 \frac{15f(x)f'(x) + 25f'(x)}{(f(x) - 3)(f(x) + 2)} \, dx$

(c) $\int_2^{10} f'(x) \ln(f(x)) \, dx$

3. For all real numbers $x$, define the function $G(x) = \int_4^x 10000 \cos \left( \frac{\pi t^2}{6} \right) \, dt$.

(a) Calculate $G'(x)$.

(b) Write out the terms of a MID(5) estimate of $G(4.5)$. (You do not need to find or approximate the numerical value of your answer.)

(c) Determine whether MID($n$) and TRAP($n$) are overestimates or underestimates of $G(4.5)$, respectively. Give a formula for, and sketch a graph of a second derivative to explain how you know this. (Feel free to use the help of Desmos or your graphing calculator.)

(d) Consider the estimates RIGHT(100), LEFT(100), TRAP(100) and MID(100) of $G(4.5)$. Suppose MID(100) $\approx -4027$ and LEFT(100) $\approx -4029$.

Recall the relationship between LEFT($n$), RIGHT($n$), and TRAP($n$). Also recall the relationship between TRAP($n$) and MID($n$) you established in part (c).

Show that RIGHT(100) $\geq -4025$. 

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