**What’s Wrong?**

Achilles and Hector, two Calculus students, are discussing their homework assignment. Achilles explains he has discovered an amazing formula. For any number $0 < x < 1$, he claims the following holds:

$$
\cdots + \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} + 1 + x + x^2 + x^3 + \cdots = 0. \tag{★}
$$

Intrigued, Hector asks him how he came up with this formula. Achilles’ step-by-step procedure is given below.

Achilles first defines the function $S(x) = \frac{1}{1-x}$.

(1) Replacing $x$ in $S(x)$ by $1/x$ and simplifying, he obtains that $S(1/x) = \frac{-x}{1-x}$.

(2) Using the result from (1), he deduces that $S(1/x) + S(x) = 1$.

Since $|x| < 1$, it is known that $S(x)$ represents the sum of the infinite geometric series:

$$
S(x) = 1 + x + x^2 + x^3 + \cdots.
$$

(3) From this and the result from (2), he derives (after simplification) his formula:

$$
\cdots + \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} + 1 + x + x^2 + x^3 + \cdots = 0.
$$

After a few seconds of thinking, Hector tells Achilles that his formula is **false**.

Explain why Hector is right, and help Achilles find his mistake.