

Web Problem #2 for Homework Assignment 8: Mortgage Payments

The specific learning goals of this web problem are for you to:

- Learn a straightforward strategy for estimating the size of a monthly mortgage payment.
- Determine how the outstanding balance on a mortgage changes over time.
- Set up a geometric series to describe the outstanding balance on a mortgage.
- Use the geometric series summation formula to work out the monthly payment on a mortgage.
- Determine the effects of (i) the size of the initial down payment, (ii) the interest rate and (iii) the length of the mortgage on the size of the monthly mortgage payment.

Your ultimate aim in this homework assignment will be to calculate and compare the monthly payments on several different mortgage options. By working through each of the questions on this homework assignment, you will have the opportunity to see how the:

- (i) Interest rate
- (ii) Size of the initial down payment
- (iii) Length of the mortgage

each affect the size of the monthly mortgage payment that a homebuyer has to make. In all of the questions on this homework, you can assume that the interest rate is fixed for the length of the mortgage and that the mortgage is an American¹ mortgage (so that interest is compounded monthly).



Figure 1: The house currently for sale in Ann Arbor, MI.

Figure 1² shows a house currently for sale in Ann Arbor, MI³. The asking price for the house is \$649,000.00. When you buy a house, you normally pay some of the cost of the house immediately (this is called the down payment) and finance the rest by taking out a mortgage. In America, mortgages normally last 15 or 30 years.

Traditionally, lenders preferred a down payment of 20% of the sale price of the property. However in recent years it has

¹ The frequency of compounding varies from country to country. In the United States interest is compounded monthly, but in Canada interest is only compounded once every six months.

² Image source:

<http://idx.prudentialsnyder.com/results.aspx?=&VIP=Yahoo!%20IDX&cc=realestate&fclose=n&newhome=n&za=and&fullnodeid=750007014&searchgeo=Ann%20Arbor.%20MI&searchtype=2&propertytype=1&sort=5&sortacdc=desc&firstrecord=60>

³ The house is located at 3329 Craig Road, Ann Arbor, MI 48103.

become more common for lenders to accept a down payment of less than 20%. (Generally speaking, however, lenders *require homebuyers to pay for private mortgage insurance to protect the lender* against any loss should the homebuyer default on the mortgage – and this insurance can be quite expensive.) Recent⁴ nominal interest rates on 15 and 30-year mortgages are given in Table 1 (below).

Duration of mortgage (years)	Nominal interest rate (%)
15	5.12
30	5.67

Table 1.

These are the interest rates that you should use for Parts (a)-(e) of this problem.

- (a) In this first part of the problem, the idea is to calculate an estimate for the monthly payment. This will help you to judge whether your answers to subsequent questions are reasonable or not. Suppose that you were interested in purchasing the house in Ann Arbor, MI, featured in Figure 1 and that you had enough money to make a 20% down payment. If you were charged no interest at all and spread the monthly mortgage payments over a 30-year period, about how much should each payment be? Is this estimate likely to be an over- or an under-estimate of the actual monthly mortgage payment that you would have to make in reality?
- (b) Let M represent the monthly mortgage payment in units of dollars. Suppose that you were interested in buying the house in Ann Arbor, MI, and had enough money to make the 20% down payment. Use Table 2 to write down symbolic expressions (involving M) for the amount of money that you would still owe each month if you took out a 30-year mortgage to finance the rest of the purchase price.

Time (months)	Amount still owed (\$)
0	
1	
2	
3	
N	

Table 2.

NOTE: You should hand in a completed version of Table 2 as your answer to this problem.

⁴ The interest rates quoted here are an average of major mortgage lenders in the United States on January 5, 2005. The interest rates may no longer apply if you are reading this document after 1/5/05.

- (c) Use the symbolic expression from Table 2 to calculate the monthly mortgage payment, assuming that you had enough money to make the 20% down payment and that you took out a 30-year (360 month) mortgage.
- (d) Now suppose that you do not have enough money to make the 20% down payment, and that your financial resources can only cover a 5% down payment. If you took out a 30-year mortgage, what would your monthly mortgage payment be? How would this lower down payment affect the total amount of money that you had to pay to the mortgage lender over the course of the loan, compared to the situation in which you were able to put down a 20% down payment?
- (e) The final variable that we will consider is the length of the loan. In the world of mortgages, the duration of the loan has three effects. First, the shorter the duration, the higher your payment as you will have less time to pay off the loan. Second, 15-year mortgages usually have a lower nominal interest rate (see Table 1) than 30-year mortgages, so you pay less interest. Third, as there are fewer months in 15 years than in 30 years there are fewer opportunities for the lender to compound interest on the outstanding balance, so the total amount of money paid over the duration of the loan is typically lower. Assuming that you can make the 20% down payment on the house in Ann Arbor, MI, and take out a 15-year mortgage to finance the rest, what will your monthly mortgage payment be? How does the total amount of money that you pay over the duration of the mortgage compare in the 15-year and 30-year (see Part (c)) cases?