## Applet Exploration: Integral Properties

For this homework question, you will need to use the applet "Integral Properties" located at http://www.jimrolf.com/integralProperties.html to answer the questions listed below. Your answers will probably need to include screen shots or other facsimiles of the graphs that you generate with the applet in order for the grader to best understand your thought processes.

**Applet information.** When the applet opens up you should see the graph of the function

$$g(x) = \int_a^x f(t)\,dt$$

on the domain $[-3,3]$. The function formula of $f(x)$ and the graph of $f(x)$ used in the above integral *have been hidden from the user*.

You may adjust the default value of $a = -3$ (the lower limit of the above integral) by dragging the slider with the mouse, by using the arrow buttons on your keyboard, or by typing a number in the text field and clicking "Graph."

If you would like to graph another function of your choosing, input the function definition in the text field next to "$h(x) =$", check the box, and click on the "Graph" button. For example, you may enter "sin(x)" (without quotes) to graph $y = \sin(x)$. There is a pull-down menu called "Function Help" in the top right corner of the applet that may help you input some special functions.

Additionally, you may want to zoom in/out via mouse clicks on the graph or slide the graph around by grabbing it with the mouse and dragging.

**Questions**

1. Describe and graph (by hand) the function $f(x)$ on the domain $[-3,3]$. In particular, you should describe where $f(x)$ is increasing/decreasing, the location of any critical points, and any function values that you can determine. As usual, it's important to explain how you arrived at these conclusions.
2. Does changing the value of the parameter *a* change your answers to question #1? Why or why not?
3. As you vary the parameter *a*, the graph of $g(x)$ appears to always cross (or touch) the *x*-axis in at least one place. In other words, $g(x) = 0$ for at least one point in the domain for all choices of *a*. Do you think this fact is specific only to the underlying hidden function $f(x)$ used in this applet? Or do you think that this would be true for every f(x)? In order to answer these questions, you should either:
   - provide an explicit example of a *new* function $f(x)$ and a value for the parameter *a* in that causes $g(x) \neq 0$ for all values of *x* on the domain $[-3,3]$; or
   - provide an explanation for why there is always a value of x between –3 and 3 such that g(x) = 0 for all values of *a* in $[-3,3]$.
4. When varying the parameter *a* from -3 to +3, the graph of $g(x)$ sometimes does not change, sometimes slides down, and sometimes slides up. Explain why this behavior occurs but yet no other characteristics of the graph change when varying *a*. *Hint:* It may be useful to consider the graph of $f(x)$ that you determined in question #1.
5. Determine which values of parameter *a* correspond to the transitions between the behaviors described in question #4. Specifically, as the parameter *a* increases, for what values of *a* is the graph of $g(x)$ sliding up, sliding down, and remaining constant? You will want to consider the graph of $f(x)$ that you determined in question #1 and provide reasoning explaining this behavior. Merely observing the value of the parameter *a* as you vary the slider is not a sufficient answer.