Most of the following exercises are modified versions of exercises from your textbook *Multivariable Calculus* by James Stewart.

15. (12.4)
   
   (a) Compute the cross product of the vectors \( \langle c, d, 0 \rangle \) and \( \langle a, b, 0 \rangle \).
   
   (b) Use the surveyor’s formula (Homework Set 0) to compute the area of the two parallelograms below.

   ![Parallelogram Diagram]

   (c) Explain the relationship between your answers to the previous two parts of this exercise. Is orientation important?

16. (12.3 and 12.4) Trout are swimming up the Blackfoot River at forty meters per hour (relative to an observer on the bank of the river) to spawn, and their density is four fish per meter cubed. John, and his sons Norman and Paul, love to fish along the Blackfoot River.\(^\text{12}\) The river runs parallel to the \( x \)-axis, its surface is parallel to the plane \( z = 0 \), and the net they are using has a rectangular opening that is one half meter wide by one half meter long with normal vector \( \mathbf{n} \). While they all put the net into the river so that \( \mathbf{n} \cdot \mathbf{j} = 0 \), each member of the family has their own special technique for using the net. Advice: Draw pictures – this is a problem about visualizing what is happening, it is not a problem about using math in clever ways.

   (a) John puts the net into the river so that its opening is perpendicular to the surface of the water (that is, \( \mathbf{n} \cdot \mathbf{k} = 0 \)), approximately how many fish does John catch in ten minutes?

   (b) Norman puts the net into the river so that its opening is parallel to the surface of the water (that is, \( \mathbf{n} \cdot \mathbf{k} = \pm 0.25 \) meters squared), approximately how many fish does Norman catch in ten minutes?

\(^\text{12}\)As in the move (and even better book) *A River Runs Through It*
(c) Paul puts the net into the river so that its opening is at an angle of thirty degrees to the surface of the water, approximately how many fish does Paul catch in ten minutes?

17. (12.4)
(a) Find three unit vectors orthogonal to \((-1/\sqrt{2}, 0, 1/\sqrt{2})\). Can you find others?
(b) Find two unit vectors which are orthogonal to both \((1, 2, 4)\) and \((-6, 1, 1)\). Can you find any others?

18. (12.5)
(a) Find the equations of the planes that are parallel to the plane \(z = 7\) and two units away from it.
(b) Find the equations of the planes that are parallel to the plane \(x + y + z = 0\) and \(4\sqrt{3}\) units away from it. (Hint: What does it mean for planes to be parallel?)

19. (12.5) Find the equation of the line consisting of those points which are equidistant from the three points \((2, 1, 3)\), \((3, -4, -1)\), and \((2, 1, -1)\).

20. (12.6) Exercises 21–28 of §12.6 of Stewart’s *Multivariable Calculus*.

21. (12.6) A cooling tower for a power plant is be constructed in the shape of a hyperboloid of one sheet, \(x^2/a^2 + y^2/b^2 - z^2/c^2 = 1\). The diameter of the circular base is 60m and the minimum diameter, 120m above the base, is 45m. Find an equation for the tower. (Hint: where is the origin?) (Bonus: Can you find the equation of a line that is in the surface?)

MatLab (a) Graph the curve with parametric equations 
\[
\mathbf{r}(t) = \left(\frac{27}{26} \sin(8t) - \frac{8}{39} \sin(18t), \frac{-27}{26} \cos(8t) + \frac{8}{39} \cos(18t), \frac{144}{65} \sin(5t)\right).
\]
(b) Show that the curve lies on the hyperboloid of one sheet \(144x^2 + 144y^2 - 25z^2 = 100\).

**Just for Fun** Don’t hand this in. You need to get from Angell Hall to the Michigan Union. It is raining and there is a strong, steady, twenty mile per hour wind blowing from the west. You have no protective rain gear, and you wish to minimize how wet you get. Should you walk, jog, or run?