Math 215
Homework Set 1: §§12.1 – 12.3
Fall 2019

The dot product has many important uses in mathematics. Among the more useful is that it allows
us to compute \( \text{proj}_w(\vec{v}) \), the “part” of vector \( \vec{v} \) that lies in the direction of vector \( \vec{w} \). For example, for a
vector \( \vec{x} = \langle a, b, c \rangle \) in \( \mathbb{R}^3 \), we have \( \text{proj}_j(\vec{x}) = \langle 0, b, 0 \rangle \). What is \( \text{proj}_k(\vec{x}) \)? What is \( \text{proj}_i(\vec{x}) \)?

Most of the following problems are modified versions of the problems from your text book *Multivariable
Calculus* by James Stewart.

12.1a. Sketch the surface in \( \mathbb{R}^3 \) represented by the equation
\[
2x - 3y + 4z + 12 = 0.
\]

12.1b. Find an equation for the set of all points equidistant from the points \( S = (-1, 2, -5) \) and \( T = (2, -2, 6) \). Describe the set.

12.1c. Describe, in words, the region in \( \mathbb{R}^3 \) represented by the inequality
\[
4x^2 + 4y^2 + 4z^2 - 36 \geq 0.
\]

12.2a. Daenerys is walking due east on the deck of the Balerion at a speed of 3 km/hr. The Balerion is
moving due south at a speed of 13 km/hr. Find Daenerys’ velocity relative to the surface of the
water.

12.2b. Suppose \( \mathbf{a} = \langle -1, 1, 2 \rangle \) and \( \mathbf{b} = \langle 2, -1, -1 \rangle \). Sketch each of the following quantities.
   (a) \( \mathbf{a} + 5\mathbf{j} \)
   (b) \( 3\mathbf{a} - 4\mathbf{b} \)

12.3a. Prove the law of cosines. (Hint: Follow the same rules as when you proved the Pythagorean theo-
rem.) Explain how the law of cosines is related to the dot product.

12.3b. Suppose \( \mathbf{u} \) and \( \mathbf{v} \) are vectors. We define \( \text{orth}_u \mathbf{v} \) to be the difference
\[
\mathbf{v} - \text{proj}_u \mathbf{v}.
\]
   (a) Show that \( \text{proj}_u \mathbf{v} \) is orthogonal to \( \text{orth}_u \mathbf{v} \).
   (b) Show that
\[
\mathbf{v} = \text{proj}_u \mathbf{v} + \text{orth}_u \mathbf{v}.
\]
   (c) Compute and sketch the vectors \( \text{proj}_u \mathbf{v} \) and \( \text{orth}_u \mathbf{v} \) for \( \mathbf{u} = \langle 3, -5, 4 \rangle \) and \( \mathbf{v} = \langle 8, 4, -6 \rangle \).

12.3c. Compute the three angles, correct to the nearest tenth of a radian, of the triangle with vertices
\(( -1, -2, -3), (3, 5, -1) \), and \((2, 4, 3) \). List your answers from least to greatest. Sketch the triangle.

12.3d. Suppose that \( \vec{u}_1 \) and \( \vec{u}_2 \) are orthogonal unit vectors in \( \mathbb{R}^2 \). That is \( |\vec{u}_1| = |\vec{u}_2| = 1 \) and \( \vec{u}_1 \cdot \vec{u}_2 = 0 \).
   (a) If \( \vec{u}_1 = \langle 1, 0 \rangle \) and \( \vec{u}_2 = \langle x, y \rangle \), then what are the possibilities for \( x \) and \( y \)?
(b) If $\vec{u}_1 = (\sqrt{2}/2, \sqrt{2}/2)$ and $\vec{u}_2 = (x, y)$, then what are the possibilities for $x$ and $y$?

(c) Suppose $\vec{v} \in \mathbb{R}^2$. Show that $\text{proj}_{\vec{u}_1}(\vec{v}) = (\vec{v} \cdot \vec{u}_1)\vec{u}_1$.

(d) Suppose $\vec{w} \in \mathbb{R}^2$. Show that $\vec{w} = (\vec{w} \cdot \vec{u}_1)\vec{u}_1 + (\vec{w} \cdot \vec{u}_2)\vec{u}_2$

12.3e. Consider the function $y = 4 \cos(t)$ where $0 \leq t \leq 5$ is measured in seconds. The following commands are entered in Matlab:

```matlab
>> t = 0:.01:5;
>> y = 4*cos(t);
```

(a) What is $y$ at 4 seconds?

(b) What will Matlab return if you type $y(4)$?

(c) Explain why your answers to the previous two questions are wildly different.