This problem set is not graded. Use it to prepare for the final exam.

1. The vector field shown below

is called a shear layer in fluid mechanics. Use Stokes theorem to argue that it has strongly nonzero curl and the divergence theorem to argue that it has zero divergence. Is the direction of the curl into the page or out of it?
Note: To apply Stokes, first note that the only component of curl that can be nonzero is the one out of the page. Next consider square loops (square because it is better for the geometry of the flow). Next argue that line integral of the vector field (circulation) around any such loop is nonzero. Is the line integral positive or negative?
Note: To apply the divergence theorem, use the same square loops but now convince yourself that the flux is zero.

2. Evaluate the flux or surface integral \( \int \int_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F} = xy\mathbf{i} + 4x^2\mathbf{j} + yz\mathbf{k} \) over the surface \( S \) given by \( z = xe^y \) with \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \). The unit normal to the surface points upward.

3. Suppose \( \mathbf{F} \) is a vector field and \( \int \int_S \text{curl}\mathbf{F} \cdot d\mathbf{S} = 1 \) where \( S \) is the surface
\[ x^2 + y^2 + z^2 = 1, \ z \geq 0, \] oriented outward. Sketch another surface for which the integral evaluates to the same number.

4. Let \( C \) be the curve of intersection of the cylinder \( x^2 + y^2 = 9 \) and the plane \( x + z = 5 \) with positive or counterclockwise sense. Let \( \mathbf{F} = xi + yj + zk \). Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) in two ways: by parameterizing the curve of intersection and using Stokes’ theorem.

5. In the vector field sketched below

![Vector Field Diagram]

use the divergence theorem to figure out if the divergence is positive or negative at \( P_1 \) and \( P_2 \).

6. Verify Stokes’ theorem for \( \mathbf{F} = -yi + xj - 2k \) and \( S \) being the cone \( z^2 = x^2 + y^2, \ 0 \leq z \leq 4 \), oriented downward.

7. Let \( \mathbf{F} = \frac{\mathbf{r}}{||\mathbf{r}||}, \) where \( \mathbf{r} = xi + yj + zk \). Find curl \( \mathbf{F} \) (also denoted \( \nabla \times \mathbf{F} \)) using Stokes’ theorem. Find \( \text{div} \mathbf{F} \) (also denoted \( \nabla \cdot \mathbf{F} \)) using the divergence theorem.

8. Find the divergence of \( \mathbf{F} = \frac{\mathbf{r}}{||\mathbf{r}||} \) using the divergence theorem.

9. Suppose a charge \( q \) is placed at the point the point \( \mathbf{r}_0 = i + j/\sqrt{2} + k/\sqrt{2} \). The electric field it generates is given by \( \mathbf{E} = q\frac{\mathbf{r} - \mathbf{r}_0}{||\mathbf{r} - \mathbf{r}_0||^3} \). Find the flux of this vector field out of the sphere \( x^2 + y^2 + z^2 = a^2 \) for \( a = 1 \) and \( a = 4 \), respectively.