Math 215: Calculus 3, HW3

Due date: 10/2 or 10/4

1. Sketch the curve \( \mathbf{r}(t) = t \mathbf{i} + 2 \sin(t) \mathbf{j} + 4 \cos(t) \mathbf{k} \) and indicate the direction along which \( t \) increases.

2. Show that the curve \( (x, y, z) = (\sin t, \cos t, \sin^2 t) \) is the curve of intersection of the surfaces \( z = x^2 \) and \( x^2 + y^2 = 1 \). Use that fact to sketch the curve.

3. Find a vector function that gives the curve of intersection of the paraboloid \( z = 4x^2 + y^2 \) and the parabolic cylinder \( y = x^2 \).

4. Two particles travel along the space curves \( \mathbf{r}_1(t) = (t, t^2, t^3) \) and \( \mathbf{r}_2(t) = (1 + 2t, 1 + 6t, 1 + 14t) \). Do the particles collide at some point in time? Do their paths intersect?

5. Sketch the plane curve \( \mathbf{r}(t) = (1 + \cos t) \mathbf{i} + (3 + \sin t) \mathbf{j} \) as well as its tangent vector \( \mathbf{r}'(t) \).

6. Find the point on the curve \( \mathbf{r}(t) = (2 \cos t, 2 \sin t, e^t), \) where \( 0 \leq t \leq \pi \), where the tangent line is parallel to the plane \( \sqrt{3}x + y = 7 \). Find the acceleration at the point.

7. At what point do the curves \( \mathbf{r}_1(t) = (t, 1 - t, 3 + t^2) \) and \( \mathbf{r}_2(t) = (3 - t, t - 2, t^2) \) intersect? Find the angle of intersection to the nearest degree.

8. Find the length of the space curve \( \mathbf{r}(t) = 12t \mathbf{i} + 8t^{3/2} \mathbf{j} + 3t^2 \mathbf{k}, \) \( 0 \leq t \leq 1 \).

9. Find the length of the intersection of the cylinder \( 4x^2 + y^2 = 4 \) and the plane \( x + y + z = 2 \) nearest to 4 decimal places (you may use a calculator).