Math 215
Homework Set 5: §§14.5 – 14.7
Fall 2019

Most of the following exercises are modified versions of exercises from your text book *Multivariable Calculus* by James Stewart.

14.5a. Because you will probably actually need to use these equations at some point in your academic career, please do Exercises 45, 52, and 53 of §14.5 of Stewart’s *Multivariable Calculus*.

14.5b. The temperature at a point \((x, y)\) on the Michigan football field is \(T(x, y)\) measured in degrees Fahrenheit. Ken Knode runs so that his position after \(t\) seconds is given by \(x = 24 + \sqrt{50 + 2t^2}\) and \(y = 14 + 3t/5\) where \(x\) and \(y\) are measured in yards. The temperature function satisfies \(T_x(34, 17) = .3\) and \(T_y(34, 17) = .1\). How fast is the temperature rising along Knode’s path after 5 seconds?

14.5c. Suppose that the equation \(H(s, t, u) = 0\) implicitly defines each of the three variables \(s, t,\) and \(u\) as functions of the other two: \(s = x(t, u), t = y(s, u),\) and \(u = z(s, t)\). If \(H\) is differentiable and \(H_s, H_t,\) and \(H_u\) are all nonzero, show that

\[1 = -\frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial t} \cdot \frac{\partial t}{\partial u}.\]

14.6a. Use the table for wave heights in Exercise 4 of §14.3 of Stewart’s *Multivariable Calculus* to estimate the value of \(D_u f(30, 20)\), where \(u = (i - j)/\sqrt{2}\). (See also Exercise 14.3a.)

14.6b. Suppose the concentration, in parts per million, of natural gas in the air is described by \(f(x, y, z) = x^2 + 3y^2 - 2z^2\). (Here \(x, y,\) and \(z\) are measured in meters.) John and Lee are at the point \((3, 5, 4)\).

(a) In which direction should John and Lee move to decrease the concentration of natural gas in the air the fastest? If they move at 5 meters/second, how fast is the concentration changing in this direction?

(b) The door John and Lee will sneak out of is located at \((2, 3, 4)\). If they move at 5 meters/second, how fast is the concentration changing in this direction?

14.6c. After copying the figure from exercise 36 of §14.6 in Stewart’s *Multivariable Calculus*, please do the exercise.

14.6d. After copying the figure from exercise 38 of §14.6 in Stewart’s *Multivariable Calculus*, please do the exercise.

14.6e. Find the points on the ellipsoid \(2x^2 + y^2 + 3z^2 = 6\) where the tangent plane is parallel to the plane \(4x + y + 6z = 7\).

14.7a. For most real world work, our observations do not allow us to develop an exact mathematical model. Thus, we look for models which “minimize” the difference between the predicted and the observed. For example, when we observe that our data appears to give a linear relation between two quantities (say temperature and the rate at which crickets chirp (see Exercise 17 in §1.2 of Stewart’s *Calculus*)) it is usually a good idea to use the method of linear regression (also called the method of least squares) to produce a mathematical model which “minimizes” the difference between the observed and the predicted. Do Exercise 59 of §14.7 in Stewart’s *Multivariable Calculus*. The

\[\text{As in the movie *Eraser*}.\]
words “minimize” and “minimizes” appear in quotes above because there may be many ways to measure the difference between the observed and the predicted; give some examples of other ways to measure this difference.

14.7b. Consider the function $f(x, y) = xy e^{-x^2 - y^2}$. Find and classify the critical points of $f$.

14.7c. You are to design a rectangular building to house the University’s art collection. Per square yard, the cost for the foundation is four times the cost of the material for the walls which is twice the cost of the material used to construct the roof. If the university has $D$ dollars to spend and the cost of the material for the roof is $d$ dollars per square foot, give the dimensions (in yards) which maximize the volume of the building.

14.7d. Find the extreme values for the function $f(x, y) = x^3 - 3x - y^3 + 12y$ on the quadrilateral whose vertices are $(-2, 3)$, $(2, 3)$, $(2, 2)$, and $(-2, -2)$. 