1. Find the directional derivative of \( x^3 + y^3 + z^3 + xyz \) at \((1, 1, 2)\) in the direction \((-2, 1, 2)\).

2. Find the directions in which the directional derivative of \( ye^{-xy} \) at \((0, 2)\) is maximum and minimum. Find the directions in which it is equal to 1.

3. Two surfaces are orthogonal if their normal lines are orthogonal. Verify that \( z^2 = x^2 + y^2 \) and \( x^2 + y^2 + z^2 = r^2 \) are orthogonal at all points of intersection.

4. The figure below shows the level curves of \( f(x, y) = 3x - x^3 - 2y^2 + y^4 \). Use it to predict the critical points of \( f \). Are the critical points maxima or minima or saddle points? Use the second derivative test to confirm your reasoning.

5. Find the maximum and minimum of \( x_1 + \cdots + x_n \) subject to \( x_1^2 + \cdots + x_n^2 = 1 \). Use Lagrange multipliers.
6. Find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.

7. Use the midpoint rule with \( m = n = 2 \) to find an approximation to \( \int \int_R xy \, dx \, dy \) where \( R = [0, 2] \times [0, 2] \).

8. Find the point on the plane \( ax + by + cz = d \) at a minimum distance from the origin using the method of Lagrange multipliers.

9. Evaluate

\[
\int \int_R \sin \pi x \cos \pi y \, dx \, dy
\]

with \( R = [0, 1/4] \times [1/4, 1/2] \).