Math 215: Calculus 3, HW6

Due date: 3/19 or 3/21

1. Evaluate $\int_0^1 \int_0^3 e^{2x+y} \, dy \, dx$.

2. Evaluate $\int \int_R (y + x/y) \, dA$ over the rectangle $R = \{(x, y) | 0 \leq x \leq 2, 1 \leq y \leq 2\}$.

3. Find the volume bounded by the surfaces $z = 16 - x^2$ and the plane $y = 5$ in the first octant.

4. Find $\int \int_R xy \, dA$ where $R$ is the region enclosed by $y = x^2$ and $y = 3x$.

5. Find $\int \int_R y^2 e^{xy} \, dA$ where $R$ is the triangle bounded by $y = x$, $y = 4$, and $x = 0$.

6. Evaluate $\int_0^1 \int_x^1 e^{x/y} \, dy \, dx$.

7. Evaluate the $\int \int_R x \, dA$ where $R$ is the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

8. Evaluate $\int_0^a \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} x^2 y \, dx \, dy$.

9. Find the mass and center of mass of a region bounded by $y = x^2$ and $x = y^2$ and of density $\rho(x, y) = \sqrt{x}$.

10. A lamina is bounded by the semicircles $y = \sqrt{4-x^2}$ and $y = \sqrt{1-x^2}$ together with segments $[-2, -1]$ and $[1, 2]$ along the $x$-axis. If its density is inversely proportional to the distance from the origin, find its center of mass.

11. Find the surface area of the cone $z^2 = x^2 + y^2$ above the disk $D \quad 0 \leq x^2 + y^2 \leq 1$.

12. Find the surface area of the hemispherical surface $x^2 + y^2 + z^2 = 1$, $z > 0$, using the $\int \int_D \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \, dA$ formula for surface area.

13. Let $D$ be a region in the $xy$ plane. Let

$$A = \int \int_D dA.$$
Let $\alpha D$ be the region in which every point $(x, y)$ in $D$ is replaced by $(\alpha x, \alpha y)$ for $\alpha > 0$. Interpret the double integral as a Riemann sum and find the area of $\alpha D$ in terms of $A$ and $\alpha$.

14. Let $\alpha D + \beta$ be the region where every point $(x, y)$ in $D$ is replaced by $(\alpha x + \beta, \alpha y + \beta)$. Find the area of $\alpha D + \beta$ in terms of $A$, the area of $D$, and $\alpha$.

15. Let $P = (a, b, h)$, $h > 0$, be a point in $\mathbb{R}^3$. If $D$ is a region in the $xy$ plane, the solid cone with base $D$ and apex $P$ is obtained by joining all points of $D$ to $P$ by line segments. If $A$ is the area of the base $D$, derive a formula for the volume of the cone in terms of $a$, $b$, $h$, and $A$. 