Most of the following exercises are modified versions of exercises from your text book *Multivariable Calculus* by James Stewart.

1. (15.4) A thin lamina is formed by considering the region inside the circle $x^2 + y^2 = 6y$ and outside the circle $x^2 + y^2 = 9$. Find the center of mass of the lamina if the density at any point (in grams per meter squared) is inversely proportional to its distance from the origin. Follow up question: why do we not care what the constant of proportionality is?

2. (15.5) Let $S$ be the portion of the plane with equation $x + y - z + 3 = 0$ that lies above the rectangle determined by $1 \leq x \leq 4$ and $2 \leq y \leq 5$
   
   (a) Sketch $S$.
   
   (b) Find the surface area of the surface $S$ by taking an appropriate cross product.
   
   (c) Find the surface area of the surface $S$ by using the material of §15.5.

3. (15.5) Let $S$ be the part of the surface $z = \frac{1 + x^2}{1 + y^2}$ that lies above the square $|x| + |y| \leq 2$.
   
   (a) Sketch $S$. (You may use Mathlab, Mathematica, etc. to do this.)
   
   (b) Set up an integral for the surface area of $S$.
   
   (c) Use Mathlab, Mathematica, etc. to evaluate the surface area of $S$, to five decimal places.

4. (15.6) Find the center of mass of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + 3y + 2z = 6$; $\rho(x, y, z) = z$.

5. (15.6) Find the center of mass of the cube given by $-a \leq x \leq a$, $-a \leq y \leq a$ and $0 \leq z \leq 2a$; $\rho(x, y, z) = x^2 + y^2 + z^2$.

6. (15.6) Sketch the region of integration for the integral

   $$\int_0^3 \int_{9-x^2}^9 \int_0^{9-y} f(x, y, z) \, dz \, dy \, dx.$$ 

   Rewrite this integral as an equivalent iterated integral in three of the five possible other orders.

7. (15.6) Find the region $E$ for which the triple integral

   $$\iiint_E (6 - 3x^2 - 2y^2 - 2z^2) \, dV$$

   is a maximum.

8. (15.7 & 15.8) Let $E$ be the solid bounded by the paraboloid $z = 9x^2 + 9y^2$ and the plane $z = b$ (here $b > 0$). If $E$ has constant density $k$, find the center of mass of $E$.

9. (15.7 & 15.8) Find the volume of one of the smaller wedges cut from a sphere of radius 27 by two planes that intersect along a diameter at an angle of $\pi/6$.

10. (15.7 & 15.8) Sketch the solid or surface described by the following equations and inequalities.
(a) $0 \leq \varphi \leq \pi/3, \rho \leq 5$
(b) $z - r^2 = 0$
(c) $4 - \rho \sin \varphi = 0$
(d) $r^2 \leq z \leq 2 - r^2$
(e) $\rho^2 - 5\rho = -6$
(f) $\pi/4 \leq \theta \leq 3\pi/4, z \leq r \leq 5$
(g) $\theta = \pi/3$.

11. (15.7 & 15.8) On one of his archeological expeditions, Henry Walton Jones has stumbled across $E$, a wonderful golden artifact.\(^{16}\) It turns out that $E$ can be modeled by looking at the region that lies inside the cylinder $x^2 + y^2 = 10$, is bounded below by the plane $z = 0$, and is bounded above by the hyperboloid $x^2 + y^2 - z^2 = 1$.

(a) Sketch $E$.
(b) Find the volume of $E$.
(c) Assuming that the density of $E$ at every point is inversely proportional to its distance from the $z$-axis, find the center of mass of $E$.


\(^{16}\)As in the movie *Raiders of the Lost Ark* or any of the many spinoffs.