1. Objectives and Expectations for Lab 1, Part A

In Part A of Lab 1, our main goal is to make use of MATLAB to study vectors, planes, and lines in the context of physical and real life problems that involve these objects. We have the following objectives.

- You will become proficient in using MATLAB for basic vector operations such as vector addition, dot product, and cross product, etc.
- You will work on vector projection using MATLAB. Projections are used quite often in data analysis.
- You will practice solving numerically physics problems in multi-dimensional settings, and be familiar with notions of acceleration, velocity, and trajectories in several dimensions.

2. Matlab Commands

2.1. \([u_1, u_2, \ldots]\) Brackets define a list in MATLAB, and it treats vectors as lists. So, to define a vector \(u = (1, 3, 5)\) in MATLAB we use the command:

\[ u = [1, 3, 5] \]

2.2. \(\text{norm}(x)\). The command \(\text{norm}(x)\) returns the \(\ell^2\)-norm \(\|x\|\) (that is, magnitude, or Euclidean length—what we think of as length) of a vector \(x = (x_1, x_2, \ldots, x_n)\), given by \(\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}\). Examples:

\[ \text{norm}([3, 4]) \]

(the result is, of course, 5); and

\[ v = [3, 3, 4, 1]; \text{norm}(v) \]

(which gives 5.9161, not \(\sqrt{35}\)—MATLAB thinks numerically).

2.3. \(\text{dot}(u,v)\). The command \(\text{dot}(u,v)\) returns the dot-product \(u \cdot v\) of two vectors \(u\) and \(v\) that are of same length. Examples:

\[ \text{dot}([3, 4], [1, 2]) \]

\[ \text{ans} = 11 \]

\[ v = [3, 3, 4, 1]; u = [0, 1, -1, 2]; \text{dot}(u,v) \]

\[ \text{ans} = 1 \]

Note that MATLAB keeps track of whether calculations make sense:

\[ v = [3, 3, 4, 1]; u = [0, 1, -1]; \text{dot}(u,v) \]

will throw the error \(\text{ans} = \text{Error}\) using \(\text{dot}\) (line XX) A and B must be same size.
2.4. **cross(u,v)**. The command `cross(u,v)` returns the cross-product \( u \times v \) of two vectors \( u \) and \( v \) that are of length 3. Examples:

```
>> cross([3,4,4],[1,2,-1])
ans =   -12    7    2
>> v = [3, 4, 0]; u= [0, 1, -1]; cross(u,v)
ans =   4   -3   -3
```

2.5. **acos(x)**. The command `acos(x)` returns \( \arccos(x) \) for a real number \( x \in [-1, 1] \). If \(-1 \leq x \leq 1\), then \( \arccos(x) \) returns real numbers in the interval \([0, \pi]\).

\[
\arccos: [-1, 1] \rightarrow [0, \pi].
\]

Otherwise, it will return a complex number. Examples:

```
>> acos(-1)
ans = 3.1416
>> acos(0.5)
ans = 1.0472
>> acos(1)
ans = 0
>> acos(1.2)
ans = 0.0000 + 0.6224i
```

In general, a complex answer is not what we want.

2.6. **plot(x,y)**. The command `plot(x,y)` takes as its arguments a vector \( x \) and another vector \( y \), and plots successive pairs of \((x,y)\) coordinates. Thus,

```
>> plot([0, 1, 2, 3, 4], [0, 1, 4, 9, 16])
```

will plot five points along the parabola \( y = x^2 \). Most functions in MATLAB are vector-aware, so that they accept a vector as an argument. Thus we can plot five points of the arccosign function with

```
>> x = [0,1,2,3,4]; plot(x, acos(x), 'o')
```

(the 'o' uses a circle as the plot symbol), and can plot a smooth function (with a blue line) with

```
>> x = [-1:0.01:1]; plot(x, acos(x), '-b')
```

2.7. **plot3(x,y,z)**. The command `plot3(x,y,z)` plots in three dimensions the curve whose \( x, y, \) and \( z \)-coordinates are given by the vectors \( x, y, \) and \( z \). Thus

```
>> t = [0:.02:12]; plot3( 2*cos(t), 2*sin(t), t, '--r' )
```

plots (with a dashed red line) a helix extending up along the \( z \)-axis.

3. **Exercises**

3.1. **Basic vector operations.** You will want to recall from the prelab the formulas for calculating a unit vector and the angle between two vectors.

**Exercise 1:** Compute a unit vector \( e_u \) in the direction of the vector \( u = \langle 1, 0.5, 3, -2 \rangle \) using the formula

\[
e_u = \frac{1}{\|u\|} u = \frac{u}{\|u\|}
\]
We use $\|u\|$ to compute the $\ell^2$-norm (Euclidean length, magnitude) $\|u\|$ of the vector $u$.

**Exercise 2:** Consider the (parametric) line equation $r(t) = \langle 1, 0, 2 \rangle + t\langle 2, 3, 1 \rangle$, $t \in \mathbb{R}$. Plot the line segment $r(t) = \langle 1, 0, 2 \rangle + t\langle 2, 3, 1 \rangle$, $0 \leq t \leq 5$ in $\mathbb{R}^3$.

**Exercise 3:** Given the intersecting lines
\[
\begin{align*}
  x &= 1 + t, \\
  y &= -1 + 2t, \\
  -2.5t, & \quad t \in \mathbb{R}
\end{align*}
\]
and
\[
\begin{align*}
  x &= 3 + -2s, \\
  y &= 1 + 1.2s, \\
  -5 + 1.2s, & \quad s \in \mathbb{R},
\end{align*}
\]
Compute the angle $0 \leq \theta \leq \pi$ (in radians) between them.

3.2. **Projections.** Recall from the prelab how to find the projection of one vector on another, and a perpendicular vector.

**Exercise 4:** Find the projection $\text{proj}_v(u)$ of $u = \langle 1, 4, 3.4, -1 \rangle$ onto the vector $v = \langle 4, -2.5, -1, 8 \rangle$.

**Exercise 5:** Find a unit vector $v$ that is perpendicular to $u = \langle 0.82, 6.45, -1.07 \rangle$.

*Hint: Use projection onto $u$. Once you find such $v$, verify numerically that $u \cdot v = 0$ (so that these vectors are perpendicular). How many such vectors $v$ are there?*

3.3. **Equations of planes.** We did not develop the equation of a plane in the prelab, but it is described in detail in your textbook, [JS].

**Exercise 6:** Given $u = \langle 6, 3.2, -1 \rangle$ and $v = \langle -3, 4, 1.84 \rangle$, find a unit vector $w$ such that $w$ is perpendicular to both $u$ and $w$. *Hint: There are only two such unit vectors. You do not need to use any projections in this case. Recall the properties of the cross-product of two vectors.*

**References**