Consider the function
\[ g(x, y) = \begin{cases} 
\frac{x^6 + y^6}{(x^6 + y^6) + (y - x^2)^2}, & (x, y) \neq (0, 0), \\
0, & (x, y) = (0, 0). 
\end{cases} \]

Observe that this function is defined for all \((x, y) \in \mathbb{R}^2\).

**Exercise 1:** Show analytically that \(\lim_{(x,y) \to (0,0)} g(x,y)\) is the same along any line \(y = ax, \ a \in \mathbb{R}\) as well as the line \(x = 0\). (\textit{Hint: Use L'Hôpital's rule.}) Verify this result by numerical evaluations using MATLAB.

**Exercise 2:** Now plot the the surface \(z = g(x, y)\). What do you see? Is \(g\) continuous at \((0, 0)\)? Explain and give a mathematical argument supporting what you see in the plot and your claim. Add graphs of representative space curves \(<x, ax, g(x, ax)>\) to your surface to illustrate your observation in Exercise 1.

**Exercise 3:** Find a curve along which the limit \(\lim_{(x,y) \to (0,0)} g(x,y)\) is different from that which you found in Exercise 2. Add this to your graph. Is \(g\) continuous?

**Exercise 4:** Find a function \(h(x, y)\) defined on all \(\mathbb{R}^2\) with \(h(0, 0) = 0\) such that \(\lim_{(x,y) \to (0,0)} h(x,y)\) is same along any line \(y = ax\) as well as the line \(x = 0\), and along any parabola \(y = bx^2\), but different along the cubic \(y = x^3\). Graph this surface and representative space curves showing the limits that you found.

**Exercise 5:** Bonus! Suppose when considering the function \(g(x, y)\) we forgot the square on the \((y - x^2)\) term (so that the denominator becomes \((x^6 + y^6) + (y - x^2))\). How do your conclusions above change?