LAB 3: LAGRANGE MULTIPLIERS, PART C
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AN APPLICATION TO ECONOMICS

Suppose that you own an electronics manufacturing company and produce
two types of circuit boards: board A and board B. To produce either type you
need some number of worker-hours and some amount of money, and these trade
off (e.g., if you spend more money on automation, you need fewer worker hours;
and in general, producing more lowers the cost per item). We generally want
to minimize the cost of production.

The constraints are (1) that we have contracts to produce some number of
each type of the circuit boards; and (2) that we have a fixed number of worker-
hours available. The first of these is modeled by the Cobb-Douglas production
model: if \( L \) is the number of labor hours spent to produce the boards and \( K \)
the dollars of capital invested,

\[
\text{(number of boards)} = k L^\alpha K^\beta,
\]

for some constants \( k \), \( \alpha \), and \( \beta \).

Let’s say that we have a contract to produce 15,000 circuit board As, and
25,000 circuit board Bs, and that the Cobb-Douglas models for these are

\[
15000 = 5 L^0.3 K^0.7 \quad \text{and} \quad 25000 = 2 L^0.4 K^0.6.
\]

Suppose you have 2000 total labor hours available, so that

\[
2000 \geq L_A + L_B.
\]

Finally, the cost of production is the capital invested in the production of
each type of board, and the cost of the labor. Suppose that each hour of labor
on circuit A costs $20, while each hour on circuit B costs $25.

(In the following you may assume that the gradients of the constraints are
linearly independent, as they are.)

**Exercise 1:** Write an equation for the total cost of production, and the
constraints. What are the variables in the optimization problem?

**Exercise 2:** Consider the case \( 2000 > L_A + L_B \). Find possible locations of
the minimum cost by using the technique from exercise 3 in part B. You
should be able to find one interior point; think carefully about how large
your guess should be.

**Exercise 3:** Now consider the boundary \( 2000 = L_A + L_B \). Find another
possible location for the minimum cost, and by using the results from the
previous problems find the minimum production cost.
**Exercise 4:** *Bonus!* We assumed the gradients of the constraints are linearly independent: show this!