Suppose that the manufacturing company now produces three types of circuit boards, and has a contract for 10,000 of board A, 20,000 of board B, and 30,000 of board C, so that the Cobb-Douglas production constraints are

\[ 10000 = 5L_A^{0.3}K_A^{0.7} \]
\[ 20000 = 2L_B^{0.4}K_B^{0.6} \]
\[ 30000 = 15L_C^{0.55}K_C^{0.45}. \]

Further, suppose that each hour of labor for boards A and B costs $20, but the cost for board C is $30/hour.

**Exercise 1:** If there is no restriction on the total labor hours, find the minimum production cost.

**Exercise 2:** Now suppose that we need to produce 40,000 units of board C. Find the minimum production cost.

**Exercise 3:** What is the relationship between your answers in exercises 1 and 2 and the value of the Lagrange multiplier for the production constraint for board C?

**Exercise 4:** Bonus! Consider the two functions \( g_1(x, y, z) = x^2 - z \) and \( g_2(x, y, z) = y^3 - z \). Find the intersection of the surfaces \( g_1(x, y, z) = 0 \) and \( g_2(x, y, z) = 0 \) and graph it in MATLAB using `plot3`. Notice where there is a sharp corner, and the minimum \( y \) value on the curve.

What happens if you use the method of Lagrange multipliers to find the extreme values of the function \( f(x, y, z) = y \) subject to the two constraints \( g_1(x, y, z) = 0 \) and \( g_2(x, y, z) = 0 \)? Are \( \nabla g_1 \) and \( \nabla g_2 \) linearly independent? Explain.