Math 215
Homework Set 1: §§13.1–13.3
Fall 2009

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

13.1.1: Find the equation of the sphere that contains the points $(-3, 0, 0)$, $(3, 4, 4)$ and $(-4, 3, 4)$, if its center lies in the xz-plane.

13.1.2: Sketch and describe in words the region in $\mathbb{R}^3$ represented by the equation

$$x^2 + y^2 + z > 2x.$$

13.1.3: Find the volume of the solid that lies inside both of the spheres

$$x^2 + y^2 + z^2 = 4$$

and

$$x^2 + y^2 + z^2 + 4x - 2y + 4z + 5 = 0.$$

13.2.1: Consider the vectors $\mathbf{a} = \langle -1, 0, 2\rangle$ and $\mathbf{b} = \langle 1, -1, 3\rangle$. Sketch each of the following quantities:

(a) $\mathbf{a} - 2\,\mathbf{b}$

(b) $-2\mathbf{a} + 3\,\mathbf{b}$

13.2.2: Problem #30 from §13.2, but take the wind speed to be 100 km/h and the the plane's airspeed to be that of a Boeing 737, about 800 km/h.

13.2.3: Problem #36 from §13.2.

13.3.1: We define the **orthogonal projection** of a vector $\mathbf{b}$ (onto another vector $\mathbf{a}$) to be $\mathrm{orth}_{\mathbf{a}}\,\mathbf{b} = \mathbf{b} - \mathrm{proj}_{\mathbf{a}}\,\mathbf{b}$.

(a) Show that $\mathrm{orth}_{\mathbf{a}}\,\mathbf{b}$ is orthogonal to $\mathbf{a}$.

(b) Under what circumstances is $\mathrm{comp}_{\mathbf{a}}\,\mathbf{b} = \mathrm{comp}_{\mathbf{b}}\,\mathbf{a}$?

(c) Under what circumstances is $\mathrm{proj}_{\mathbf{a}}\,\mathbf{b} = \mathrm{proj}_{\mathbf{b}}\,\mathbf{a}$?

(d) Under what circumstances is $\mathrm{orth}_{\mathbf{a}}\,\mathbf{b} = \mathrm{orth}_{\mathbf{b}}\,\mathbf{a}$?

13.3.2: Use scalar projection to show that the distance from a point (x_1, y_1) to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$