Math 215
Homework Set 4: §§15.5–15.7
Fall 2009

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

15.5.1: Problem #37 from §15.5. Be sure to carefully explain how you obtain your answer.

15.5.2: Problem #54 from §15.5.

15.6.1: Let $g(x, y) = x^2 - y^2 + 4xy$. Find the equation(s) of the level curve with $g(x, y) = 4$. [Hint: you can solve for y if you want to get an explicit formula for the curve(s).] Sketch or plot with *Maple* the level curve. Find the gradient vector $\nabla g(1, 1)$. Use this to find the tangent line to the level curve at the point $(1, 1)$. Add the tangent line and gradient vector to your sketch of the level curve.

15.6.2: At what point(s), if any, on the paraboloid $y = x^2 + z^2$ is(are) the tangent plane(s) parallel to the plane $x + 2y + 3z = 1$?

15.6.3: Show that every normal line to the sphere $x^2 + y^2 + z^2 = r^2$ passes through the center of the sphere.

15.7.1: Problem #24 from §15.7.

15.7.2: Find an equation of the plane that passes through the point $(1, 2, 3)$ and cuts off the smallest volume in the first octant.

M.3: *Maple* problem 3. The plane $\frac{x}{3} + \frac{y}{2} + z = 3$ intersects the cylinder $x^2 + y^2 = 4$ in an ellipse.

(a) Find a parametric representation $\mathbf{r}(t)$ of the ellipse.

(b) Use *Maple* to plot the cylinder, plane, and $\mathbf{r}(t)$ together (you may want to use the `implicitplot3d` command to graph the cylinder). (Include your graph in your homework solutions.)

(c) From your graph, estimate the location of the highest point on the ellipse.

(d) Plot the projection of the ellipse in the xy-plane, and use the `display` command to show the gradient vector field and the projection of the ellipse on the same graph. Explain how this picture shows the location of the highest point on the ellipse.